Supplementary Material for

Aggregate Implications of Firm Heterogeneity: A Nonparametric Analysis of Monopolistic Competition Trade Models

Not Intended for Publication

by Rodrigo Adao, Costas Arkolakis, and Sharat Ganapati

A Supplementary Theory Appendix: Extensions

This appendix presents five extensions of our baseline framework. In Section A.1, we extend our model to include heterogeneous firms in multiple sectors whose production function uses multiple factors and sector-specific inputs. In Section A.2, we relax the assumption of full support in the distribution of entry potentials to allow for zero trade flows between countries. In Section A.3, we incorporate import tariffs into trade costs and government revenue. Section A.4 extends our baseline framework to allow firms to produce multiple products. Finally, Section A.5 relaxes the CES assumption in our framework by allowing for a general class of demand functions with a single aggregator.

A.1 Multi-Sector, Multi-Factor Heterogeneous Firm Model with Input-Output Links

In this section, we extend our baseline framework to allow for firm heterogeneity in a model with multiple sectors, multiple factors of production, and input-output linkages. Our specification of the model can be seen as a generalziation of the formulation in Costinot and Rodriguez-Clare (2013).

A.1.1 Environment

The world economy is constituted of countries with multiple sectors indexed by s. Each country has a representative household that inelastically supplies $\bar{L}_{i,f}$ units of multiple factors of production indexed by f.

Preferences. The representative household in country j has CES preferences over the composite good of multiple sectors, $s = 1, ...S$:

$$U_j = \left[\sum_s \gamma_j^s \left(Q_j^k\right)^{\frac{\lambda_j - 1}{\lambda_j}}\right]^{\frac{\lambda_j - 1}{\lambda_j}}.$$

Given the price of the sectoral composite goods, the share of spending on sector s is

$$c_j^s = \gamma_j^s \left(\frac{P_j^s}{P_j}\right)^{1-\lambda_j} \tag{A.1}$$

where the consumption price index is

$$P_j = \left[\sum_k \gamma_j^s \left(P_j^k\right)^{1-\lambda_j}\right]^{\frac{1}{1-\lambda_j}}. \tag{A.2}$$

Sectoral final composite good. In each sector s of country j, there is a perfectly competitive market for a non-tradable final good whose production uses different varieties of the tradable input good in sector s:

$$Q_j^s = \left(\sum_i \int_{\Omega_{ij}^s} \left(\bar{b}_{ij}^s b_{ij}^s(\omega)\right)^{\frac{1}{\sigma^s}} \left(q_{ij}^s(\omega)\right)^{\frac{\sigma^s - 1}{\sigma^s}} \, d\omega\right)^{\frac{\sigma^s}{\sigma^s - 1}}$$

where $\sigma_j^s > 1$ and Ω_{ij}^s is the set of sector s's varieties of intermediate goods produced in country i available in country j.

The demand of country j by variety ω of sector s in country i is

$$q_{ij}^{s}(\omega) = \left(\bar{b}_{ij}^{s} b_{ij}^{s}(\omega)\right) \left(\frac{p_{ij}^{s}(\omega)}{P_{j}^{s}}\right)^{-\sigma^{s}} \frac{E_{j}^{s}}{P_{j}^{s}}$$

where E_j^s is the total spending of country j in sector s.

Because the market for the composite sectoral good is competitive, the price is the CES price index of intermediate inputs:

$$\left(P_{j}^{s}\right)^{1-\sigma^{s}} = \sum_{i} \int_{\Omega_{ij}^{s}} \left(\bar{b}_{ij}^{s} b_{ij}^{s}(\omega)\right) \left(p_{ij}^{s}(\omega)\right)^{1-\sigma^{s}} \, d\omega. \tag{A.3}$$

Sectoral intermediate good. In sector s of country i, there is a representative competitive firm that produces a non-traded sectoral intermediate good using different factors and the non-traded composite final good of different sectors. The production function is

$$q_{i}^{s} = \left[\alpha_{i}^{s} \left(L_{i}^{s}\right)^{\frac{\mu_{i}^{s}-1}{\mu_{i}^{s}}} + \left(1-\alpha_{i}^{s}\right) \left(M_{i}^{s}\right)^{\frac{\mu_{i}^{s}-1}{\mu_{i}^{s}}}\right]^{\frac{\mu_{i}^{s}}{\mu_{i}^{s}-1}},$$

where

$$L_{i}^{s} = \left[\sum_{f} \beta_{i}^{s,f} \left(L_{i}^{s,f}\right)^{\frac{\eta_{i}^{s}-1}{\eta_{i}^{s}}}\right]^{\frac{\eta_{i}^{s}}{\eta_{i}^{s}-1}} \quad \text{and} \quad M_{i}^{s} = \left[\sum_{k} \theta_{i}^{ks} \left(Q_{i}^{k}\right)^{\frac{\kappa_{i}^{s}-1}{\kappa_{i}^{s}}}\right]^{\frac{\kappa_{i}^{s}-1}{\kappa_{i}^{s}}}.$$

Zero profit implies that the price of the sectoral intermediate good is

$$\bar{p}_{i}^{s} = \left[\alpha_{i}^{s} \left(W_{i}^{s}\right)^{1-\mu_{i}^{s}} + \left(1-\alpha_{i}^{s}\right) \left(V_{i}^{s}\right)^{1-\mu_{i}^{s}}\right]^{\frac{1}{1-\mu_{i}^{s}}}, \tag{A.4}$$

where

$$W_{i}^{s} = \left[\sum_{f} \beta_{i}^{s,f} \left(w_{i}^{f}\right)^{1-\eta_{i}^{s}}\right]^{\frac{1}{1-\eta_{i}^{s}}} \quad \text{and} \quad V_{i}^{s} = \left[\sum_{k} \theta_{i}^{ks} \left(P_{i}^{k}\right)^{1-\kappa_{i}^{s}}\right]^{\frac{1}{1-\kappa_{i}^{s}}}. \tag{A.5}$$

The share of total production cost in sector s spent on factor f and input k are given by

$$l_{i}^{s,f} = \beta_{i}^{s,f} \left(\frac{w_{i}^{f}}{W_{i}^{s}}\right)^{1-\eta_{i}^{s}} \alpha_{i}^{s} \left(\frac{W_{i}^{s}}{\bar{p}_{i}^{s}}\right)^{1-\mu_{i}^{s}} \quad \text{and} \quad m_{i}^{ks} = \theta_{i}^{ks} \left(\frac{P_{i}^{k}}{V_{i}^{s}}\right)^{1-\kappa_{i}^{s}} \left(1-\alpha_{i}^{s}\right) \left(\frac{V_{i}^{s}}{\bar{p}_{i}^{s}}\right)^{1-\mu_{i}^{s}}. \tag{A.6}$$

Production of traded intermediate varieties ω. Assume that sector s has a continuum of monopolistic firms that produce output using only a non-tradable input q_i^s. In order to sell q in market j, variety ω of country i faces a cost function given by

$$C_{ij}(\omega, q) = \bar{p}_{i}^{s} \frac{\tau_{ij}^{s}(\omega)}{a_{i}^{s}(\omega)} \frac{\bar{\tau}_{ij}^{s}}{\bar{a}_{i}^{s}} q + \bar{p}_{i}^{s} \bar{f}_{ij}^{s} f_{ij}^{s}(\omega)$$

where $\bar{p}_i^s$ is the price of the non-tradable input q_i^s in country i.

Given this production technology, the optimal price is $p_{ij}^s(\omega) = \frac{\sigma_j^s}{\sigma_j^s - 1} \frac{\tau_{ij}^s(\omega)}{a_i^s(\omega)} \frac{\bar{\tau}_{ij}^s}{\bar{a}_i^s} \bar{p}_i^s$ and the associated revenue is

$$R_{ij}^{s}(\omega) = r_{ij}^{s}(\omega) \bar{r}_{ij}^{s} \left[\left(\frac{\bar{p}_{i}^{s}}{P_{j}^{s}}\right)^{1-\sigma^{s}} E_{j}^{s}\right] \tag{A.7}$$

where

$$r_{ij}^s(\omega) \equiv b_{ij}^s(\omega)\left(\frac{\tau_{ij}^s(\omega)}{a_i^s(\omega)}\right)^{1-\sigma^s} \quad \text{and} \quad \bar{r}_{ij} \equiv \bar{b}_{ij}^s\left(\frac{\sigma^s}{\sigma^s-1}\frac{\bar{\tau}_{ij}^s}{\bar{a}_i^s}\right)^{1-\sigma^s}. \tag{A.8}$$

Firm ω of country i chooses to enter a foreign market j if, and only if, $\pi_{ij}^s(\omega) = (1/\sigma_j^s)R_{ij}^s(\omega) - \bar{p}_i^s \bar{f}_{ij}^s f_{ij}^s(\omega) \geq 0$. This condition determines the set of firms from country i that operate in sector s of country j:

$$\omega \in \Omega_{ij}^s \quad \Leftrightarrow e_{ij}^s(\omega) \geq \sigma^s \frac{\bar{f}_{ij}^s}{\bar{r}_{ij}^s}\left[\left(\frac{\bar{p}_i^s}{P_j^s}\right)^{\sigma^s}\frac{P_j^s}{E_j^s}\right], \tag{A.9}$$

where

$$e_{ij}^s(\omega) \equiv \frac{r_{ij}^s(\omega)}{f_{ij}^s(\omega)}. \tag{A.10}$$

Entry of traded intermediate varieties ω. Firms in sector s of country i can create a new variety by spending $\bar{F}_i^s$ units of the non-tradable sectoral input q_i^s. In this case, they take a draw of the variety characteristics from an arbitrary distribution:

$$v_i(\omega) \equiv \left\{a_i^s(\omega), b_{ij}^s(\omega), \tau_{ij}^s(\omega), f_{ij}^s(\omega)\right\}_j \sim G_i^s(v). \tag{A.11}$$

In equilibrium, free entry implies that N_i^s firms pay the fixed cost of entry in exchange for an ex-ante expected profit of zero,

$$\sum_j E\left[\max\left\{\pi_{ij}^s(\omega);\ 0\right\}\right] = \bar{p}_i^s \bar{F}_i^s. \tag{A.12}$$

Market clearing. We follow Dekle et al. (2008) by allowing for a set of exogenous transfers. Thus, the spending on goods of sector s by country i is

$$E_i^s = c_j^s(w_i L_i + T_i) + \sum_k m_i^{sk}\left(\bar{p}_i^k q_i^k\right). \tag{A.13}$$

The market clearing conditions for factor f in country i is

$$w_i^f \bar{L}_i^f = \sum_s l_i^{s,f}\left(\bar{p}_i^s q_i^s\right). \tag{A.14}$$

Since all the revenue of the sectoral intermediate good comes from sales to the firms producing the varieties ω, we have that

$$\frac{\bar{p}_i^s q_i^s}{N_i^s} = \underbrace{\sum_j\left(1-\frac{1}{\sigma^s}\right)Pr[\omega \in \Omega_{ij}^s]E[R_{ij}^s(\omega)|\omega \in \Omega_{ij}^s]}_{\text{final good production}} + \underbrace{\sum_j \bar{p}_i^s \bar{f}_{ij}^s Pr[\omega \in \Omega_{ij}^s]E[f_{ij}^s(\omega)|\omega \in \Omega_{ij}^s]}_{\text{fixed-cost of entering markets}} + \underbrace{\bar{p}_i^s \bar{F}_i^s}_{\text{fixed cost of entry}}.$$

The free entry condition in (A.12) implies that

$$\bar{p}_i^s \bar{F}_i^s = \sum_j E\left[\max\left\{\pi_{ij}^s(\omega);\ 0\right\}\right] = \sum_j Pr[\omega \in \Omega_{ij}^s]\left(\frac{1}{\sigma^s}E[R_{ij}^s(\omega)|\omega \in \Omega_{ij}^s] - \bar{p}_i^s \bar{f}_{ij}^s E[f_{ij}^s(\omega)|\omega \in \Omega_{ij}^s]\right).$$

Thus, $\bar{p}_i^s q_i^s = \sum_j N_i^s Pr[\omega \in \Omega_{ij}^s] E[R_{ij}^s(\omega) | \omega \in \Omega_{ij}^s]$ and, therefore,

$$\bar{p}_i^s q_i^s = \sum_j \bar{r}_{ij}^s \left[\left(\frac{\bar{p}_i^s}{P_j^s} \right)^{1-\sigma^s} E_j^s \right] \left[\int_{\omega \in \Omega_{ij}^s} r_{ij}^s(\omega)\, d\omega \right]. \tag{A.15}$$

Equilibrium. Given the distribution in (A.11), the equilibrium is P_i, $\{\Omega_{ij}^s\}_{j,s}$, $\{P_i^s, N_i^s, \bar{p}_i^s, W_i^s, V_i^s, \bar{p}_i^s q_i^s, E_i^s, c_i^s\}_s$, $\{m_i^{sk}\}_{k,s}$, $\{l_i^{s,f}\}_{f,s}$ and $\{w_i^f\}_f$ for all i that satisfy equations (A.2), (A.9). (A.3), , (A.12), (A.4), (A.5), (A.6), (A.13), (A.1), (A.15), (A.14).

A.1.2 Extensive and Intensive margin of Firm-level Export

We now turn to the characterization of the bilateral levels of entry and sales in each sector. As before, we consider the marginal distribution of $\left(r_{ij}^s(\omega), e_{ij}^s(\omega)\right)$ implied by G_i^s, which can be decomposed without loss of generality as

$$r_{ij}^s(\omega) \sim H_{ij}^{r,s}(r|e), \qquad \text{and} \qquad e_{ij}^s(\omega) \sim H_{ij}^{e,s}(e), \tag{A.16}$$

where $H_{ij}^{e,s}$ has full support in $\mathbb{R}_+$.

Extensive margin of firm-level exports. The share of firms in sector s of country i serving market j is $n_{ij}^s = Pr\left[\omega \in \Omega_{ij}^s\right]$. We define $\epsilon_{ij}^s(n) \equiv \left(H_{ij}^{e,s}\right)^{-1}(1-n)$ such that

$$\ln \epsilon_{ij}^s(n_{ij}^s) = \ln\left(\sigma^s \bar{f}_{ij}^s / \bar{r}_{ij}^s\right) + \ln\left(\bar{p}_i^s\right)^{\sigma^s} - \ln E_j^s \left(P_j^s\right)^{\sigma^s - 1}. \tag{A.17}$$

Thus, we obtain a sector-specific version of the relationship between the function of the share of firms from i selling in j and the linear combination of exogenous bilateral trade shifters and endogenous outcomes in the origin and destination markets.

Intensive margin of firm-level exports. The average revenue of firms from country i in country j is $\bar{x}_{ij}^s \equiv E\left[R_{ij}^s(\omega) | \omega \in \Omega_{ij}^s\right]$. Define the average revenue potential of exporters when $n_{ij}^s\%$ of $i's$ firms in sector s export to j as $\rho_{ij}^s\left(n_{ij}^s\right) \equiv \frac{1}{n_{ij}^s} \int_0^{n_{ij}^s} \tilde{\rho}_{ij}^s(n)\, dn$ where $\tilde{\rho}_{ij}^s(n) \equiv E[r | e = \epsilon_{ij}^s(n)]$ is the average revenue potential in quantile n of the entry potential distribution. Using the transformation $n = 1 - H_{ij}^{e,s}(e)$ such that $e = \epsilon_{ij}^s(n)$ and $dH_{ij}^{e,s}(e) = -dn$, we can follow the same steps as in the baseline model to show that

$$\ln \bar{x}_{ij}^s - \ln \rho_{ij}^s(n_{ij}^s) = \ln\left(\bar{r}_{ij}^s\right) + \ln\left(\bar{p}_i^s\right)^{1-\sigma^s} + \ln E_j^s \left(P_j^s\right)^{\sigma^s - 1}. \tag{A.18}$$

Thus, we obtain a sector-specific version of the relationship between the composition-adjusted per-firm sales and a linear combination of exogenous bilateral trade shifters and endogenous outcomes in the origin and destination markets.

A.1.3 General Equilibrium

We now write the equilibrium conditions in terms of $\rho_{ij}^s(n)$ and $\epsilon_{ij}^s(n)$. We start by writing the price index P_j^s in (A.3) in terms of $\rho_{ij}^s(n)$. Using the expression for $p_{ij}^s(\omega)$ and (A.3), we have that $(P_j^s)^{1-\sigma^s} = \sum_i \bar{r}_{ij}^s \left(\bar{p}_i^s\right)^{1-\sigma^s} \int_{\Omega_{ij}^s} r_{ij}^s(\omega)\, d\omega$. Since $\int_{\Omega_{ij}^s} r_{ij}^s(\omega)\, d\omega = N_i^s Pr[\omega \in \Omega_{ij}^s] E[r | \omega \in \Omega_{ij}^s] = N_i^s n_{ij}^s \rho_{ij}^s(n_{ij}^s)$, we can write P_j^s as

$$(P_j^s)^{1-\sigma^s} = \sum_i \bar{r}_{ij}^s \left(\bar{p}_i^s\right)^{1-\sigma^s} \rho_{ij}^s(n_{ij}^s) n_{ij}^s N_i^s. \tag{A.19}$$

We then turn to the free entry condition in (A.12). Following the same steps as in Appendix A.1, it is straight forward to show that

$$\mathbb{E}\left[\max\left\{\pi_{ij}^s(\omega);\ 0\right\}\right] = \frac{1}{\sigma^s} n_{ij}^s \bar{x}_{ij}^s - \bar{p}_i^s \bar{f}_{ij}^s \int_0^{n_{ij}} \frac{\tilde{\rho}_{ij}^s(n)}{\epsilon_{ij}^s(n)}\ dn,$$

which implies that the free entry condition in (A.12) is equivalent to

$$\sigma^s \bar{p}_i^s F_i^s = \sum_j n_{ij}^s \bar{x}_{ij}^s - \sum_j \left(\sigma^s \bar{p}_i^s \bar{f}_{ij}^s\right) \int_0^{n_{ij}} \frac{\tilde{\rho}_{ij}^s(n)}{\epsilon_{ij}^s(n)}\ dn.$$

Notice that the summation of (A.17) and (A.18) implies that $\ln\left(\sigma^s \bar{p}_i^s \bar{f}_{ij}^s\right) = \ln \bar{x}_{ij}^s - \ln \bar{\rho}_{ij}^s(n_{ij}^s) + \ln \bar{\epsilon}_{ij}^s(n_{ij}^s)$. Thus, following again the same steps as in Appendix A.1, it is straight forward to show that

$$\sigma^s \bar{p}_i^s \bar{F}_i^s = \sum_j n_{ij}^s \bar{x}_{ij}^s \left(1 - \frac{\int_0^{n_{ij}^s} \frac{\tilde{\rho}_{ij}^s(n)}{\epsilon_{ij}^s(n)}\ dn}{\int_0^{n_{ij}^s} \frac{\tilde{\rho}_{ij}^s(n)}{\epsilon_{ij}^s(n_{ij}^s)}\ dn}\right). \tag{A.20}$$

Finally, we established above that $\bar{p}_i^s q_i^s = \sum_j N_i^s Pr[\omega \in \Omega_{ij}^s] E[R_{ij}^s(\omega) | \omega \in \Omega_{ij}^s]$. Since $n_{ij}^s \equiv Pr[\omega \in \Omega_{ij}^s]$ and $\bar{x}_{ij}^s \equiv E[R_{ij}^s(\omega) | \omega \in \Omega_{ij}^s]$, then

$$\bar{p}_i^s q_i^s = \sum_j N_i^s n_{ij}^s \bar{x}_{ij}^s. \tag{A.21}$$

This implies that, given $\left\{\{L_i^f\}_f, \{F_i^s, \alpha_i^s, \gamma_i^s, \eta_i^s, \mu_i^s, \kappa_i^s\}_s, \{\bar{r}_{ij}^s, \bar{f}_{ij}^s\}_{j,s}, \{\beta_i^{s,f}\}_{f,s}, \{\theta_i^{sk}\}_{k,s}, \lambda_i\right\}_i$, an equilibrium vector $\left\{P_i, \{n_{ij}^s, \bar{x}_{ij}^s\}_{j,s}, \{P_i^s, N_i^s, \bar{p}_i^s, W_i^s, V_i^s \bar{p}_i^s q_i^s, E_i^s, c_i^s\}_s, \{m_i^{sk}\}_{k,s}, \{l_i^{s,f}\}_{f,s}, \{w_i^f\}_f\right\}_i$ satisfies the following conditions.

1. The extensive and intensive margins of firm-level sales, n_{ij}^s and $\bar{x}_{ij}^s$, satisfy (A.17) and (A.18) for all s, i and j.
2. The price of the final sectoral good P_j^s is given by (A.19). The final consumption good price P_i is given by (A.2).
3. The number of entrants in sector s of country i N_i^s satisfies the free entry condition in (A.20).
4. The price of the the intermediate sector good $\bar{p}_i^s$ is given by (A.4) where W_i^s and V_i^s are given by (A.5).
5. The total revenue of the intermediate sectoral good $\bar{p}_i^s q_i^s$ is given by (A.21).
6. Spending on the final sectoral good E_i^s is (A.13) with final consumption spending share c_i^s given by (A.1) and the intermediate consumption spending share m_i^{sk} given by (A.6).
7. Factor price w_i^s implies that the factor market clearing in (A.14) holds with l_i^f given by (A.6).

A.1.4 Nonparametric Counterfactual Predictions

We now use the equilibrium characterization above to compute counterfactual changes in aggregate outcomes using the functions $\epsilon_{ij}^s(n)$ and $\rho_{ij}^s(n)$. As in our baseline model, this implies that we do not need any parametric restrictions in the distribution of firm heterogeneity G_i. The implications of firm heterogeneity for the model's aggregate counterfactual predictions are summarized by $\epsilon_{ij}^s(n)$ and $\rho_{ij}^s(n)$.

The extensive and intensive margins of firm-level sales in (A.17) and (A.18) imply that

$$\ln \frac{\epsilon_{ij}^s(n_{ij}^s \hat{n}_{ij}^s)}{\epsilon_{ij}^s(n_{ij}^s)} = \ln\left(\hat{\bar{f}}_{ij}^s / \hat{\bar{r}}_{ij}^s\right) + \ln\left(\hat{\bar{p}}_i^s\right)^{\sigma^s} - \ln \hat{E}_j^s \left(\hat{P}_j^s\right)^{\sigma^s - 1}. \tag{A.22}$$

$$\ln \hat{\bar{x}}_{ij}^s - \ln \frac{\rho_{ij}^s(n_{ij}^s \hat{n}_{ij}^s)}{\rho_{ij}^s(n_{ij}^s)} = \ln\left(\hat{\bar{r}}_{ij}^s\right) + \ln\left(\hat{\bar{p}}_i^s\right)^{1-\sigma^s} + \ln \hat{E}_j^s \left(\hat{P}_j^s\right)^{\sigma^s - 1}. \tag{A.23}$$

The price of the final sectoral good P_j^s in (A.19) implies that

$$(\hat{P}_j^s)^{1-\sigma^s} = \sum_i \frac{\bar{x}_{ij}^s n_{ij}^s N_i^s}{E_j^s} \left(\hat{\bar{r}}_{ij}^s \left(\hat{\bar{p}}_i^s\right)^{1-\sigma^s} \frac{\rho_{ij}^s(n_{ij}^s \hat{n}_{ij}^s)}{\rho_{ij}^s(n_{ij}^s)} \hat{n}_{ij}^s \hat{N}_i^s\right).$$

Let $x_{ij}^s \equiv \bar{x}_{ij}^s n_{ij}^s N_i^s / E_j^s = X_{ij}^s / \left(\sum_o X_{oj}^s\right)$ be the spending share of country j on country i. Thus,

$$(\hat{P}_j^s)^{1-\sigma^s} = \sum_i x_{ij}^s \left(\hat{\bar{r}}_{ij}^s \left(\hat{\bar{p}}_i^s\right)^{1-\sigma^s} \frac{\rho_{ij}^s(n_{ij}^s \hat{n}_{ij}^s)}{\rho_{ij}^s(n_{ij}^s)} \hat{n}_{ij}^s \hat{N}_i^s\right). \tag{A.24}$$

The final consumption good price P_i in (A.2) implies that

$$\hat{P}_j^{1-\lambda_j} = \sum_k c_j^s \left(\hat{P}_j^k\right)^{1-\lambda_j}. \tag{A.25}$$

The free entry condition in (A.20) implies that

$$\hat{\bar{p}}_i^s \hat{\bar{F}}_i^s \sum_j n_{ij}^s \bar{x}_{ij}^s \left(1 - \frac{\int_0^{n_{ij}^s} \frac{\tilde{\rho}_{ij}^s(n)}{\epsilon_{ij}^s(n)}\, dn}{\int_0^{n_{ij}} \frac{\tilde{\rho}_{ij}^s(n)}{\epsilon_{ij}^s(n_{ij})}\, dn}\right) = \sum_j n_{ij}^s \bar{x}_{ij}^s (\hat{n}_{ij}^s \hat{\bar{x}}_{ij}^s) \left(1 - \frac{\int_0^{n_{ij}^s \hat{n}_{ij}^s} \frac{\tilde{\rho}_{ij}^s(n)}{\epsilon_{ij}^s(n)}\, dn}{\int_0^{n_{ij}^s \hat{n}_{ij}^s} \frac{\tilde{\rho}_{ij}^s(n)}{\epsilon_{ij}^s(n_{ij}^s \hat{n}_{ij}^s)}\, dn}\right). \tag{A.26}$$

The price of the the intermediate sector good $\bar{p}_i^s$ in (A.4) implies that

$$\hat{\bar{p}}_i^s = \left[\tilde{\alpha}_i^s \left(\hat{W}_i^s\right)^{1-\mu_i^s} + (1 - \tilde{\alpha}_i^s)\left(\hat{V}_i^s\right)^{1-\mu_i^s}\right]^{\frac{1}{1-\mu_i^s}}, \tag{A.27}$$

where $\tilde{\alpha}_i^s$ is the share of labor in total cost of sector s in country i.

From (A.5), $\hat{W}_i^s$ and $\hat{V}_i^s$ are given by

$$\hat{W}_i^s = \left[\sum_f l_i^{s,f} \left(\hat{w}_i^f\right)^{1-\eta_i^s}\right]^{\frac{1}{1-\eta_i^s}} \quad \text{and} \quad \hat{V}_i^s = \left[\sum_k m_i^{ks} \left(\hat{P}_i^k\right)^{1-\kappa_i^s}\right]^{\frac{1}{1-\kappa_i^s}}. \tag{A.28}$$

The total revenue of the intermediate sectoral good $\bar{p}_i^s q_i^s$ in (A.21) implies

$$\widehat{\bar{p}_i^s q_i^s} = \sum_j x_{ij}^s \hat{N}_i^s \hat{n}_{ij}^s \hat{\bar{x}}_{ij}^s.$$

Let $\iota_i \equiv w_i L_i / (w_i L_i + T_i)$. Spending on the final sectoral good E_i^s in (A.13) implies that

$$\hat{E}_i^s = \hat{c}_j^s \frac{w_i L_i + T_i}{E_j} \left(\iota_i \hat{w}_i + (1 - \iota_i)\hat{T}_i\right) + \sum_k \frac{\tilde{M}_i^{ks}}{E_i^s} \widehat{\bar{p}_i^s q_i^s}.$$

where $\tilde{M}_i^{ks}$ is the value of intermediate sales of sector k to s in country i.

The final consumption spending share c_i^s in (A.1) implies that

$$\hat{c}_j^s = \left(\frac{\hat{P}_j^s}{\hat{P}_j}\right)^{1-\lambda_j}. \tag{A.29}$$

The intermediate consumption spending share m_i^{sk} in (A.6) implies that

$$\hat{m}_i^{ks} = \left(\frac{\hat{P}_i^k}{\hat{V}_i^s}\right)^{1-\kappa_i^s} \left(\frac{\hat{V}_i^s}{\hat{\bar{p}}_i^s}\right)^{1-\mu_i^s}. \tag{A.30}$$

The labor spending share l_i^f in (A.6).

$$\hat{l}_i^{s,f} = \left(\frac{\hat{w}_i^f}{\hat{W}_i^s}\right)^{1-\eta_i^s} \left(\frac{\hat{W}_i^s}{\hat{\bar{p}}_i^s}\right)^{1-\mu_i^s}. \tag{A.31}$$

The factor market clearing in (A.14) implies that

$$\hat{w}_i^f = \sum_s \zeta_i^{f,s} \hat{l}_i^{s,f} \left(\widehat{\bar{p}_i^s q_i^s}\right) \tag{A.32}$$

where $\zeta_i^{f,s}$ is the share of factor f income coming from sector s in country i.

Thus, the system (A.22)–(A.31) determines the counterfactual predictions in the model.

A.2 Allowing for zero bilateral trade

In this section, we extend our baseline framework to allow zero trade flows between two countries.

A.2.1 Environment

Consider the same environment described in Section 2.1.

A.2.2 Extensive and Intensive Margin of Firm Export

As in our baseline, we consider the distribution of $(r_{ij}(\omega), e_{ij}(\omega))$ implied by $G_i(.)$:

$$r_{ij}(\omega) \sim H_{ij}^r(r|e), \qquad \text{and} \qquad e_{ij}(\omega) \sim H_{ij}^e(e). \tag{A.33}$$

To allow for zero trade flows, we follow Helpman et al. (2008) by allows the support of the entry potential distribution to be bounded. Specifically, assume that $H_{ij}(e)$ has full support over $[0, \bar{e}_{ij}]$.

Extensive margin of firm-level exports. Recall that $n_{ij} \equiv Pr[\omega \in \Omega_{ij}]$ where Ω_{ij} is given by (5). It implies that

$$n_{ij} = \begin{cases} 1 - H_{ij}^e(e_{ij}^*) & \text{if} \quad e_{ij}^* \leq \bar{e}_{ij} \\ 0 & \text{if} \quad e_{ij}^* > \bar{e}_{ij} \end{cases} \qquad \text{where} \quad e_{ij}^* \equiv \sigma \frac{\bar{f}_{ij}}{\bar{r}_{ij}} \left[\left(\frac{w_i}{P_j}\right)^{\sigma} \frac{P_j}{E_j}\right].$$

Let us now define the extensive margin function as

$$\tilde{\epsilon}_{ij}(n_{ij}) \equiv \begin{cases} \left(H^e_{ij}\right)^{-1}(1-n) & \text{if} \quad n > 0 \\ \bar{e}_{ij} & \text{if} \quad n = 0 \end{cases}.$$

Using this definition and the expression for n_{ij} above, we get that

$$\tilde{\epsilon}_{ij}(n_{ij}) = \min\left\{e^*_{ij}, \bar{e}_{ij}\right\}.$$

We then define $\epsilon_{ij}(n) \equiv \tilde{\epsilon}_{ij}(n_{ij})/\bar{e}_{ij}$ and $\tilde{f}_{ij} \equiv \bar{f}_{ij}/\bar{e}_{ij}$. Then,

$$\ln \epsilon_{ij}(n_{ij}) = \min\left\{\ln\left(\sigma \tilde{f}_{ij}\bar{r}_{ij}\right) + \ln\left(w_i^{\sigma}\right) - \ln\left(E_j P_j^{\sigma-1}\right), 0\right\}. \tag{A.34}$$

Intensive margin of firm-level exports. Conditional on $n_{ij} > 0$, we now compute the average revenue in j:

$$\bar{x}_{ij} = \bar{r}_{ij}\left[\left(\frac{w_i}{P_j}\right)^{1-\sigma} E_j\right] \frac{1}{n_{ij}} \int_{e^*_{ij}}^{\underline{e}_{ij}} E[r|e]\ dH^e_{ij}(e).$$

We consider the transformation $n = 1 - H^e_{ij}(e)$ such that $e = \epsilon_{ij}(n)$ and $dH_{ij}(e) = -dn$. By defining $\tilde{\rho}_{ij}(n) \equiv E[r|e = \epsilon_{ij}(n)]$ and $\rho_{ij}(0) = 0$,

$$\ln \bar{x}_{ij} - \ln \rho_{ij}(n_{ij}) = \ln\left(\bar{r}_{ij}\right) + \ln\left(w_i^{1-\sigma}\right) + \ln\left(E_j P_j^{\sigma-1}\right). \tag{A.35}$$

A.2.3 General Equilibrium

We now write the equilibrium conditions in terms of $\rho_{ij}(.)$ and $\epsilon_{ij}(.)$. Since $x_{ij} = \bar{x}_{ij} n_{ij} N_i / E_j$ and $\sum_i x_{ij} = 1$, the expression above immediately implies that

$$P_j^{1-\sigma} = \sum_{i:n_{ij}>0} \bar{r}_{ij}\left(w_i\right)^{1-\sigma} \bar{\rho}_{ij}(n_{ij})\left(n_{ij}N_i\right) \tag{A.36}$$

We then turn to the free entry condition in (7). Following the same steps as in Appendix A.1, it is straight forward to show that

$$\sigma w_i F_i = \sum_j n_{ij}\bar{x}_{ij} - \sum_j \left(\sigma w_i \bar{f}_{ij}\right) \int_0^{n_{ij}} \frac{\tilde{\rho}_{ij}(n)}{\epsilon_{ij}(n)}\ dn.$$

For $n_{ij} > 0$, the ratio of (A.34) and (A.35) implies that $\sigma \bar{f}_{ij} w_i = \bar{x}_{ij}\epsilon_{ij}(n_{ij})/\rho_{ij}(n_{ij})$. Thus,

$$\sigma w_i F_i = \sum_{j:n_{ij}>0} n_{ij}\bar{x}_{ij}\left(1 - \frac{\int_0^{n_{ij}} \frac{\tilde{\rho}_{ij}(n)}{\epsilon_{ij}(n)}\ dn}{\int_0^{n_{ij}} \frac{\tilde{\rho}_{ij}(n)}{\epsilon_{ij}(n_{ij})}\ dn}\right). \tag{A.37}$$

Following the same steps as in Appendix A.1, it is straight forward to show that

$$w_i L_i = \sum_{j:n_{ij}>0} N_i n_{ij}\bar{x}_{ij}. \tag{A.38}$$

Thus, given $\left\{\bar{L}_i, \bar{F}_i, \{\bar{r}_{ij}, \tilde{f}_{ij}\}_j\right\}_i$, an equilibrium vector $\{\{n_{ij}, \bar{x}_{ij}\}_j, P_i, N_i, E_i, w_i\}_i$ satisfies the following conditions.

1. The extensive and intensive margins of firm-level sales, n_{ij} and $\bar{x}_{ij}$, satisfy (A.34) and (A.35) for all i and j.
2. For all i, the price index is given by (A.36).
3. For all i, free entry is given by (A.37).
4. For all i, total spending, E_i, satisfies (8).
5. For all i, the labor market clearing is given by (A.38).

A.2.4 Nonparametric Counterfactual Predictions

We now use the equilibrium characterization above to compute counterfactual changes in aggregate outcomes using the functions $\epsilon_{ij}(n)$ and $\rho_{ij}(n)$. We further assume that, in bilateral pairs for which initially $n_{ij} = 0$, we still have that $n'_{ij} = 0$. Thus, (A.34) implies that

$$\frac{\epsilon_{ij}(n_{ij}\hat{n}_{ij})}{\epsilon_{ij}(n_{ij})} = \frac{\hat{\bar{f}}_{ij}}{\hat{\bar{r}}_{ij}} \left[\left(\frac{\hat{w}_i}{\hat{P}_j} \right)^{\sigma} \frac{\hat{P}_j}{\hat{E}_j} \right] \quad \text{for} \quad n_{ij} > 0 \tag{A.39}$$

$$n'_{ij} = 0 \quad \text{for} \quad n_{ij} = 0. \tag{A.40}$$

The intensive margin equation remains the same:

$$\hat{\bar{x}}_{ij} = \hat{\bar{r}}_{ij} \frac{\rho_{ij}(n_{ij}\hat{n}_{ij})}{\rho_{ij}(n_{ij})} \left[\left(\frac{\hat{w}_i}{\hat{P}_j} \right)^{1-\sigma} \hat{E}_j \right] \quad \text{for} \quad n_{ij} > 0. \tag{A.41}$$

The price index equation in (A.36) implies that

$$\hat{P}_j^{1-\sigma} = \sum_{i:n_{ij}>0} x_{ij} \left(\hat{\bar{r}}_{ij} \frac{\rho_{ij}(n_{ij}\hat{n}_{ij})}{\rho_{ij}(n_{ij})} \hat{w}_i^{1-\sigma} \hat{n}_{ij} \hat{N}_i \right). \tag{A.42}$$

The spending equation in (8) implies that

$$\hat{E}_i = \iota_i \left(\hat{w}_i \hat{\bar{L}}_i \right) + (1 - \iota_i) \hat{\bar{T}}_i, \tag{A.43}$$

The labor market clearing condition in (A.38) implies

$$\hat{w}_i \hat{\bar{L}}_i = \sum_{j:n_{ij}>0} y_{ij} \left(\hat{N}_i \hat{n}_{ij} \hat{\bar{x}}_{ij} \right). \tag{A.44}$$

The free entry condition in (A.37) implies that

$$\hat{w}_i \sum_{j:n_{ij}>0} n_{ij} \bar{x}_{ij} \left(1 - \frac{\int_0^{n_{ij}} \frac{\tilde{\rho}_{ij}(n)}{\epsilon_{ij}(n)}\, dn}{\int_0^{n_{ij}} \frac{\tilde{\rho}_{ij}(n)}{\epsilon_{ij}(n_{ij})}\, dn} \right) = \sum_{j:n_{ij}>0} n_{ij} \bar{x}_{ij} (\hat{n}_{ij} \hat{\bar{x}}_{ij}) \left(1 - \frac{\int_0^{n_{ij}\hat{n}_{ij}} \frac{\tilde{\rho}_{ij}(n)}{\epsilon_{ij}(n)}\, dn}{\int_0^{n_{ij}\hat{n}_{ij}} \frac{\tilde{\rho}_{ij}(n)}{\epsilon_{ij}(n_{ij}\hat{n}_{ij})}\, dn} \right). \tag{A.45}$$

Thus, the system (A.39)–(A.45) determines the counterfactual predictions in the model.

A.3 Model with Import Tariffs

In this section, we follow Costinot and Rodriguez-Clare (2013) to extend our baseline framework to allow for import tariffs.

A.3.1 Environment

We assume that country j charges an ad-valorem tariff of t_{ij} such that the total trade costs between country i and j is $\bar{\tau}_{ij}(1+t_{ij})$. We consider a monopolistic competitive environment in which firms maximize profits given the demand in (1). For firm ω of country i, the optimal price in market j is $p_{ij}(\omega) = \frac{\sigma}{\sigma-1}\frac{\bar{\tau}_{ij}(1+t_{ij})w_i}{\bar{a}_i}\frac{\tau_{ij}(\omega)}{a_i(\omega)}$ with an associated revenue of

$$R_{ij}(\omega) = \bar{r}_{ij} r_{ij}(\omega)\left[\left(\frac{w_i}{P_j}\right)^{1-\sigma} E_j\right], \tag{A.46}$$

where

$$r_{ij}(\omega) \equiv b_{ij}(\omega)\left(\frac{\tau_{ij}(\omega)}{a_i(\omega)}\right)^{1-\sigma} \quad \text{and} \quad \bar{r}_{ij} \equiv \bar{b}_{ij}\left(\frac{\sigma}{\sigma-1}\frac{\bar{\tau}_{ij}(1+t_{ij})}{\bar{a}_i}\right)^{1-\sigma}. \tag{A.47}$$

The firm's entry decision depends on the profit generated by the revenue in (A.46), $\sigma^{-1}(1+t_{ij})^{-1}R_{ij}(\omega)$, and the fixed-cost of entry, $w_i\bar{f}_{ij}f_{ij}(\omega)$. Specifically, firm ω of i enters j if, and only if, $\pi_{ij}(\omega) = \sigma^{-1}(1+t_{ij})^{-1}R_{ij}(\omega) - w_i\bar{f}_{ij}f_{ij}(\omega) \geq 0$. This yields the set of firms from i selling in j:

$$\omega \in \Omega_{ij} \quad \Leftrightarrow e_{ij}(\omega) \geq \sigma(1+t_{ij})\frac{\bar{f}_{ij}}{\bar{r}_{ij}}\left[\left(\frac{w_i}{P_j}\right)^{\sigma}\frac{P_j}{E_j}\right], \tag{A.48}$$

where

$$e_{ij}(\omega) \equiv \frac{r_{ij}(\omega)}{f_{ij}(\omega)}. \tag{A.49}$$

The aggregate trade flows (including tariff) is still given by

$$X_{ij} = \int_{\omega\in\Omega_{ij}} R_{ij}(\omega)\, d\omega. \tag{A.50}$$

As before, free entry implies that N_i satisfies

$$\sum_j E\left[\max\{\pi_{ij}(\omega);\ 0\}\right] = w_i\bar{F}_i. \tag{A.51}$$

Market clearing. As shown by Costinot and Rodriguez-Clare (2013), the country's spending now also includes the tariff revenue:

$$E_i = w_i\bar{L}_i + \bar{T}_i + \sum_i \frac{t_{ij}}{1+t_{ij}}X_{ij}. \tag{A.52}$$

Now a fraction $t_{ij}/(1+t_{ij})$ of total revenue goes to the government of country j. So, labor in country i only receive a fraction $1/(1+t_{ij})$ of the sales revenue. Thus, $w_iL_i = (1+t_{ij})^{-1}\int_{\omega\in\Omega_{ij}} R_{ij}(\omega)\, d\omega$ and, by (A.47),

$$w_i\bar{L}_i = \frac{\bar{r}_{ij}}{1+t_{ij}}\left(\frac{w_i}{P_j}\right)^{1-\sigma} E_j\left[\int_{\omega\in\Omega_{ij}} r_{ij}(\omega)\, d\omega\right]. \tag{A.53}$$

A.3.2 Extensive and Intensive Margin of Firm Export

Using the same definitions of the baseline model, expression (A.48) yields

$$\ln \epsilon_{ij}(n_{ij}) = \ln\left(\sigma(1+t_{ij})\bar{f}_{ij}/\bar{r}_{ij}\right) + \ln\left(w_i^{\sigma}\right) - \ln\left(E_j P_j^{\sigma-1}\right). \tag{A.54}$$

Again, following the same steps of the baseline model, equation (A.46) implies the same intensive margin equation:

$$\ln \bar{x}_{ij} - \ln \rho_{ij}(n_{ij}) = \ln\left(\bar{r}_{ij}\right) + \ln\left(w_i^{1-\sigma}\right) + \ln\left(E_j P_j^{\sigma-1}\right). \tag{A.55}$$

A.3.3 General Equilibrium

Part 1. To derive the labor market clearing condition notice that there are three sources of demand for labor: production of goods, fixed-cost of entering a market and fixed-cost of creating a variety. Thus,

$$w_i\bar{L}_i = \sum_j N_i Pr[\omega\in\Omega_{ij}]\left(1-\frac{1}{\sigma}\right)\frac{1}{1+t_{ij}}E\left[R_{ij}\left(\omega\right)|\omega\in\Omega_{ij}\right] + \sum_j N_i Pr[\omega\in\Omega_{ij}]w_i\bar{f}_{ij}E\left[f_{ij}\left(\omega\right)|\omega\in\Omega_{ij}\right] + N_i w_i \bar{F}_i$$

From the free entry condition, we know that

$$w_i\bar{F}_i = \sum_j \mathbb{E}\left[\max\left\{\pi_{ij}(\omega);\ 0\right\}\right] = \sum_j Pr[\omega\in\Omega_{ij}]\left(\frac{1}{\sigma}\frac{1}{1+t_{ij}}E\left[R_{ij}\left(\omega\right)|\omega\in\Omega_{ij}\right] - w_i\bar{f}_{ij}E\left[f_{ij}\left(\omega\right)|\omega\in\Omega_{ij}\right]\right),$$

which implies that

$$w_i\bar{L}_i = \sum_j N_i Pr[\omega\in\Omega_{ij}]E\left[R_{ij}\left(\omega\right)|\omega\in\Omega_{ij}\right]\frac{1}{1+t_{ij}}.$$

Thus, since $\bar{x}_{ij} \equiv E\left[R_{ij}\left(\omega\right)|\omega\in\Omega_{ij}\right]$ and $n_{ij} = Pr[\omega\in\Omega_{ij}]$, this immediately implies that

$$w_i\bar{L}_i = \sum_j \frac{N_i n_{ij}\bar{x}_{ij}}{1+t_{ij}}. \tag{A.56}$$

Part 2. Since $p_{ij}\left(\omega\right) = \frac{\sigma}{\sigma-1}\frac{\bar{\tau}_{ij}w_i}{\bar{a}_i}\frac{\tau_{ij}(\omega)}{a_i(\omega)}$, the expression for $P_j^{1-\sigma}$ in (2) implies that

$$P_j^{1-\sigma} = \sum_i\left[\bar{b}_{ij}\left(\frac{\sigma}{\sigma-1}\frac{(1+t_{ij})\bar{\tau}_{ij}}{\bar{a}_i}\right)^{1-\sigma}\right]\left(w_i^{1-\sigma}\right)\int_{\Omega_{ij}}\left(b_{ij}(\omega)\right)\left(\frac{\tau_{ij}(\omega)}{a_i(\omega)}\right)^{1-\sigma}\ d\omega$$

Using the definitions in (4), we can write this expression as

$$P_j^{1-\sigma} = \sum_i \bar{r}_{ij}\left(w_i^{1-\sigma}\right)\int_{\Omega_{ij}} r_{ij}\left(\omega\right)\ d\omega$$

Notice that $\int_{\Omega_{ij}} r_{ij}(\omega)\ d\omega = N_i Pr[\omega\in\Omega_{ij}]E[r|\omega\in\Omega_{ij}] = N_i n_{ij}\rho_{ij}(n_{ij})$. This immediately yields

$$P_j^{1-\sigma} = \sum_i \bar{r}_{ij} w_i^{1-\sigma}\rho_{ij}(n_{ij})n_{ij}N_i. \tag{A.57}$$

Part 3. We start by writing

$$\begin{aligned}\mathbb{E}\left[\max\left\{\pi_{ij}(\omega);\ 0\right\}\right] &= Pr[\omega\in\Omega_{ij}]E\left[\pi_{ij}\left(\omega\right)|\omega\in\Omega_{ij}\right]+Pr[\omega\notin\Omega_{ij}]0\\ &= Pr[\omega\in\Omega_{ij}]\left(\tfrac{1}{\sigma}\tfrac{1}{1+t_{ij}}E\left[R_{ij}\left(\omega\right)|\omega\in\Omega_{ij}\right]-w_i\bar{f}_{ij}E\left[f_{ij}\left(\omega\right)|\omega\in\Omega_{ij}\right]\right)\\ &= n_{ij}\left(\tfrac{1}{\sigma}\tfrac{1}{1+t_{ij}}\bar{x}_{ij}-w_i\bar{f}_{ij}E\left[r_{ij}(\omega)/e_{ij}(\omega)|\omega\in\Omega_{ij}\right]\right)\end{aligned}$$

where the second equality follows from the expression for $\pi_{ij}(\omega)=\frac{1}{\sigma}\frac{1}{1+t_{ij}}R_{ij}(\omega)-w_i\bar{f}_{ij}f_{ij}\left(\omega\right)$, and the third equality follows from the definitions of $\bar{x}_{ij}\equiv E\left[R_{ij}\left(\omega\right)|\omega\in\Omega_{ij}\right]$ and $e_{ij}(\omega)\equiv r_{ij}(\omega)/f_{ij}(\omega)$.

By defining $e^*_{ij}\equiv\sigma(1+t_{ij})\frac{\bar{f}_{ij}}{\bar{r}_{ij}}\left[\left(\frac{w_i}{P_j}\right)^{\sigma}\frac{P_j}{E_j}\right]$, we can write

$$E\left[r_{ij}(\omega)/e_{ij}(\omega)|\omega\in\Omega_{ij}\right]=\int_{e^*_{ij}}^{\infty}\frac{1}{e}\left[\int_0^{\infty}rdH^r_{ij}\left(r|e\right)\right]\frac{dH^e(e)}{1-H^e(e^*_{ij})}$$

Consider the transformation $n=1-H_{ij}(e)$ such that $e=\bar{\epsilon}_{ij}(n)$. In this case, $dH_{ij}(e)=-dn$ and $n_{ij}=1-H_{ij}(e^*_{ij})$, which implies that

$$E\left[r_{ij}(\omega)/e_{ij}(\omega)|\omega\in\Omega_{ij}\right]=\frac{1}{n_{ij}}\int_0^{n_{ij}}\frac{\tilde{\rho}_{ij}(n)}{\epsilon_{ij}(n)}dn.$$

Thus,

$$\mathbb{E}\left[\max\left\{\pi_{ij}(\omega);\ 0\right\}\right]=\frac{1}{\sigma}\frac{1}{1+t_{ij}}n_{ij}\bar{x}_{ij}-w_i\bar{f}_{ij}\int_0^{n_{ij}}\frac{\tilde{\rho}_{ij}(n)}{\epsilon_{ij}(n)}\ dn.$$

Thus, the free entry condition is

$$\sigma w_i\bar{F}_i=\sum_j\frac{n_{ij}\bar{x}_{ij}}{1+t_{ij}}-\sum_j\left(\sigma w_i\bar{f}_{ij}\right)\int_0^{n_{ij}}\frac{\tilde{\rho}_{ij}(n)}{\epsilon_{ij}(n)}\ dn. \tag{A.58}$$

Notice that the summation of (A.54) and (A.55) implies that

$$\ln\left(\sigma(1+t_{ij})w_i\bar{f}_{ij}\right)=\ln\bar{x}_{ij}-\ln\rho_{ij}(n_{ij})+\ln\epsilon_{ij}(n_{ij})$$

which yields

$$\sigma w_i\bar{F}_i=\sum_j\frac{n_{ij}\bar{x}_{ij}}{1+t_{ij}}-\sum_j\frac{n_{ij}\bar{x}_{ij}}{1+t_{ij}}\frac{\epsilon_{ij}(n_{ij})}{\rho_{ij}(n_{ij})}\int_0^{n_{ij}}\frac{\tilde{\rho}_{ij}(n)}{\epsilon_{ij}(n)}\ dn.$$

Using the market clearing condition in (OA.1), we have that

$$\frac{1}{N_i}=\sigma\frac{\bar{F}_i}{\bar{L}_i}+\sum_j\frac{n_{ij}\bar{x}_{ij}}{(1+t_{ij})w_i\bar{L}_i}\frac{\epsilon_{ij}(n_{ij})}{\rho_{ij}(n_{ij})}\int_0^{n_{ij}}\frac{\tilde{\rho}_{ij}(n)}{\epsilon_{ij}(n)}\ dn. \tag{A.59}$$

which immediately yields equation (OA.7).

Part 4. Equation (A.52) implies that

$$E_i=w_i\bar{L}_i+\bar{T}_i+\sum_j\frac{t_{ji}}{1+t_{ji}}(N_jn_{ji}\bar{x}_{ji}). \tag{A.60}$$

Thus, given $\left\{\bar{L}_i,\bar{F}_i,\{t_{ij},\bar{r}_{ij},\bar{f}_{ij}\}_j\right\}_i$, an equilibrium vector $\left\{\{n_{ij},\bar{x}_{ij}\}_j,P_i,N_i,E_i,w_i\right\}_i$ satisfies the following conditions.

1. The extensive and intensive margins of firm-level sales, n_{ij} and $\bar{x}_{ij}$, satisfy (A.54) and (A.55) for all i and j.
2. For all i, the price index is given by (A.57).
3. For all i, free entry is given by (A.59).
4. For all i, total spending, E_i, satisfies (A.60).
5. For all i, the labor market clearing is given by (A.56).

A.3.4 Nonparametric Counterfactual Predictions

We now use the equilibrium characterization above to compute counterfactual changes in aggregate outcomes using the functions $\epsilon_{ij}(n)$ and $\rho_{ij}(n)$.

From (A.54) and (A.55),

$$\frac{\epsilon_{ij}(n_{ij}\hat{n}_{ij})}{\epsilon_{ij}(n_{ij})} = (\widehat{1+t_{ij}})\frac{\hat{\bar{f}}_{ij}}{\hat{\bar{r}}_{ij}}\frac{\hat{w}_i^{\sigma}}{\hat{E}_j\hat{P}_j^{\sigma-1}}. \tag{A.61}$$

$$\hat{\bar{x}}_{ij} = \frac{\rho_{ij}(n_{ij}\hat{n}_{ij})}{\rho_{ij}(n_{ij})}\hat{\bar{r}}_{ij}\frac{\hat{E}_j\hat{P}_j^{\sigma-1}}{\hat{w}_i^{\sigma-1}} \tag{A.62}$$

From (A.57),

$$\hat{P}_j^{1-\sigma} = \sum_i x_{ij}\hat{\bar{r}}_{ij}\hat{w}_i^{1-\sigma}\frac{\rho_{ij}(n_{ij}\hat{n}_{ij})}{\rho_{ij}(n_{ij})}\hat{n}_{ij}\hat{N}_i. \tag{A.63}$$

From (A.59),

$$N_i\hat{N}_i = \left[\sigma\frac{\bar{F}_i}{\bar{L}_i}\frac{\hat{\bar{F}}_i}{\hat{\bar{L}}_i} + \sum_j \frac{n_{ij}\bar{x}_{ij}}{(1+t_{ij})w_i\bar{L}_i}\frac{\hat{n}_{ij}\hat{\bar{x}}_{ij}}{(\widehat{1+t_{ij}})\hat{w}_i\hat{\bar{L}}_i}\frac{\int_0^{n_{ij}\hat{n}_{ij}}\frac{\tilde{\rho}_{ij}(n)}{\epsilon_{ij}(n)}\,dn}{\int_0^{n_{ij}\hat{n}_{ij}}\frac{\tilde{\rho}_{ij}(n)}{\epsilon_{ij}(n_{ij}\hat{n}_{ij})}\,dn}\right]^{-1}$$

Using (A.59) to substitute for $\sigma\frac{\bar{F}_i}{\bar{L}_i}$,

$$\hat{N}_i = \left[\left(1-\sum_j y_{ij}\frac{\int_0^{n_{ij}}\frac{\tilde{\rho}_{ij}(n)}{\epsilon_{ij}(n)}\,dn}{\int_0^{n_{ij}}\frac{\tilde{\rho}_{ij}(n)}{\epsilon_{ij}(n_{ij})}\,dn}\right)\frac{\hat{\bar{F}}_i}{\hat{\bar{L}}_i} + \sum_j y_{ij}\frac{\hat{n}_{ij}\hat{\bar{x}}_{ij}}{(\widehat{1+t_{ij}})\hat{w}_i\hat{\bar{L}}_i}\frac{\int_0^{n_{ij}\hat{n}_{ij}}\frac{\tilde{\rho}_{ij}(n)}{\epsilon_{ij}(n)}\,dn}{\int_0^{n_{ij}\hat{n}_{ij}}\frac{\tilde{\rho}_{ij}(n)}{\epsilon_{ij}(n_{ij}\hat{n}_{ij})}\,dn}\right]^{-1}. \tag{A.64}$$

where $y_{ij} = \frac{X_{ij}}{(1+t_{ij})w_i\bar{L}_i}$ is the share of income in i from sales to j.

From (A.60),

$$\hat{E}_i = \iota_i\hat{w}_i\hat{\bar{L}}_i + \vartheta_i\hat{\bar{T}}_i + \sum_j\left(\frac{t_{ji}}{1+t_{ji}}\frac{\hat{t}_{ji}}{(\widehat{1+t_{ji}})}\right)\frac{X_{ji}}{E_i}(\hat{N}_j\hat{n}_{ji}\hat{\bar{x}}_{ji}). \tag{A.65}$$

where $\iota_i \equiv Y_i/E_i$ and $\vartheta_i \equiv \bar{T}_i/E_i$.

Thus, the system (A.61)–(A.65) determines the counterfactual predictions in the model.

A.4 Multi-product Firms

In this section, we extend our framework to incorporate multi-product firms.

A.4.1 Environment

Demand. We maintain the assumption that each country j has a representative household that inelastically supplies $\bar{L}_j$ units of labor. The demand for variety ω from country i is

$$q_{ij}(\omega) = \bar{b}_{ij} \left(\frac{p_{ij}(\omega)}{P_j} \right)^{-\sigma} \frac{E_j}{P_j}, \tag{A.66}$$

where, in market j, E_j is the total spending, $p_{ij}(\omega)$ is the price of variety ω of country i, and P_j is the CES price index,

$$P_j^{1-\sigma} = \sum_i \int_{\Omega_{ij}^v} \bar{b}_{ij} \left(p_{ij}(\omega) \right)^{1-\sigma} \, d\omega, \tag{A.67}$$

and Ω_{ij}^v is the set of varieties produced in country i that are sold in country j.

Production. We consider a monopolistic competitive environment. Each firm η can choose how many varieties to sell in each market. In order to operate in market j, the firm must pay a fixed entry cost $w_i \bar{f}_{ij} f_{ij}(\eta)$. Conditional on entry, selling N varieties entails a labor cost of $w_i \frac{1}{1+1/\alpha} N^{1+1/\alpha}$. For every variety, the firm then has a unit production cost of $w_i \frac{\tau_{ij}(\eta)}{a_i(\eta)} \frac{\bar{\tau}_{ij}}{\bar{a}_i}$.

For each variety ω of firm η from country i, the optimal price in market j is $p_{ij}(\omega) = \frac{\sigma}{\sigma-1} \frac{\bar{\tau}_{ij} w_i}{\bar{a}_i} \frac{\tau_{ij}(\eta)}{a_i(\eta)}$ with an associated revenue of

$$R_{ij}^N(\eta) = \bar{r}_{ij}^N r_{ij}^N(\eta) \left[\left(\frac{w_i}{P_j} \right)^{1-\sigma} E_j \right], \tag{A.68}$$

where

$$r_{ij}^N(\eta) \equiv \left(\frac{\tau_{ij}(\eta)}{a_i(\eta)} \right)^{1-\sigma} \quad \text{and} \quad \bar{r}_{ij}^N \equiv \bar{b}_{ij} \left(\frac{\sigma}{\sigma - 1} \frac{\bar{\tau}_{ij}}{\bar{a}_i} \right)^{1-\sigma}. \tag{A.69}$$

The firm then decides how many varieties to sell by solving the following problem:

$$\max_N \frac{1}{\sigma} R_{ij}^N(\eta) N - w_i \frac{1}{1+1/\alpha} N^{1+1/\alpha},$$

which implies that

$$N_{ij}(\eta) = \left(\frac{1}{\sigma} \frac{R_{ij}^N(\eta)}{w_i} \right)^{\alpha}. \tag{A.70}$$

Thus, firm sales are

$$R_{ij}(\eta) = N_{ij}(\eta) R_{ij}^N(\eta) = \frac{1}{\sigma^\alpha w_i^\alpha} \left(\bar{r}_{ij}^N r_{ij}^N(\eta) \right)^{1+\alpha} \left[\left(\frac{w_i}{P_j} \right)^{1-\sigma} E_j \right]^{1+\alpha}.$$

To simplify the notation, conditional on entering market j, the sales of firm η can be written as

$$R_{ij}(\eta) = \bar{r}_{ij} r_{ij}(\eta) w_i^{1-(1+\alpha)\sigma} \left(E_j P_j^{\sigma-1} \right)^{1+\alpha} \tag{A.71}$$

$$r_{ij}(\eta) \equiv \left(r_{ij}^N(\eta) \right)^{1+\alpha} \quad \text{and} \quad \bar{r}_{ij} \equiv \frac{1}{\sigma^\alpha} \left(\bar{r}_{ij}^N \right)^{1+\alpha}. \tag{A.72}$$

Conditional on entering market j, the firm's profit in that market is

$$\begin{aligned}\pi_{ij}(\eta) \quad &= \quad N_{ij}(\eta)\tfrac{1}{\sigma}R_{ij}^{N}(\eta) - w_i\tfrac{1}{1+1/\alpha}N_{ij}(\eta)^{1+1/\alpha} - w_i\bar{f}_{ij}f_{ij}(\eta)\\ &\left(\tfrac{1}{\sigma}\tfrac{R_{ij}^{N}(\eta)}{w_i}\right)^{\alpha}\tfrac{1}{\sigma}R_{ij}^{N}(\eta) - w_i\tfrac{1}{1+1/\alpha}\left(\tfrac{1}{\sigma}\tfrac{R_{ij}^{N}(\eta)}{w_i}\right)^{1+\alpha} - w_i\bar{f}_{ij}f_{ij}(\eta)\\ &\tfrac{1}{(1+\alpha)\sigma}\tfrac{1}{\sigma^{\alpha}w_i^{\alpha}}\left(R_{ij}^{N}(\eta)\right)^{1+\alpha} - w_i\bar{f}_{ij}f_{ij}(\eta)\end{aligned}$$

and, therefore,

$$\pi_{ij}(\eta) = \frac{1}{(1+\alpha)\sigma}\bar{r}_{ij}r_{ij}(\eta)\,w_i^{1-(1+\alpha)\sigma}\left(E_jP_j^{\sigma-1}\right)^{1+\alpha} - w_i\bar{f}_{ij}f_{ij}(\eta). \tag{A.73}$$

The cost of variety creation is $C_{ij}^{V}(\eta) = w_i\frac{1}{1+1/\alpha}\left(\frac{1}{\sigma}\frac{R_{ij}^{N}(\eta)}{w_i}\right)^{1+\alpha}$, which can be written as

$$C_{ij}^{V}(\eta) = \frac{\alpha}{(1+\alpha)\sigma}\bar{r}_{ij}r_{ij}(\eta)\,w_i^{1-(1+\alpha)\sigma}\left(E_jP_j^{\sigma-1}\right)^{1+\alpha}. \tag{A.74}$$

Firm η of i sells in j if, and only if profits are positive, $\pi_{ij}(\eta) \geq 0$. This yields the set of firms of country i operating in j:

$$\eta \in \Omega_{ij} \quad \Leftrightarrow e_{ij}(\eta) \geq (1+\alpha)\sigma\frac{\bar{f}_{ij}}{\bar{r}_{ij}}\frac{w_i^{(1+\alpha)\sigma}}{\left(E_jP_j^{\sigma-1}\right)^{1+\alpha}} \tag{A.75}$$

where

$$e_{ij}(\eta) \equiv \frac{r_{ij}(\eta)}{f_{ij}(\eta)}. \tag{A.76}$$

Entry. An entrant firm pays a fixed labor cost $\bar{F}_i$ to draw its type from an arbitrary distribution:

$$v_i(\eta) \equiv \{a_i(\eta), \tau_{ij}(\eta), f_{ij}(\eta)\}_j \sim G_i^{v}(v), \tag{A.77}$$

In equilibrium, N_i firms pay the fixed cost of entry in exchange for an ex-ante expected profit of zero. The free entry implies that

$$\sum_j E\left[\max\left\{\pi_{ij}(\eta);\ 0\right\}\right] = w_i\bar{F}_i. \tag{A.78}$$

Market clearing. We follow Dekle et al. (2008) by introducing exogenous international transfers, so that spending is

$$E_i = w_i\bar{L}_i + \bar{T}_i, \quad \sum_i \bar{T}_i = 0. \tag{A.79}$$

Since labor is the only factor of production, labor income in i equals the total revenue of firms from i: $w_iL_i = \sum_j \int R_{ij}(\eta)d\eta$. Given the expression in (A.71),

$$w_i\bar{L}_i = \sum_j \bar{r}_{ij}w_i^{1-(1+\alpha)\sigma}\left(E_jP_j^{\sigma-1}\right)^{1+\alpha}\int_{\eta\in\Omega_{ij}} r_{ij}(\eta)\,d\eta. \tag{A.80}$$

Equilibrium. Given the arbitrary distribution in (A.77), the equilibrium is defined as the vector $\{P_i, \{\Omega_{ij}\}_j, N_i, E_i, w_i\}_i$ satisfying equations (A.67), (A.75), (A.78), (A.79), (A.80) for all i.

A.4.2 Extensive and Intensive Margin of Firm Exports

The extensive and intensive margin of firm exports follows the single-product case. We now use the definitions of entry and revenue potentials to characterize firm-level entry and sales in different markets in general equilibrium. We consider the CDF of $(r_{ij}(\eta), e_{ij}(\eta))$ implied by G_i. We assume that

$$r_{ij}(\eta) \sim H^r_{ij}(r|e), \qquad \text{and} \qquad e_{ij}(\eta) \sim H^e_{ij}(e). \tag{A.81}$$

Extensive margin of firm-level exports. The share of firms of country i serving market j is $n_{ij} = Pr[\eta \in \Omega_{ij}]$. Defining $\epsilon_{ij}(n) \equiv \left(H^e_{ij}\right)^{-1}(1-n)$, equation (A.75) yields

$$\ln \epsilon_{ij}(n_{ij}) = \ln\left((1+\alpha)\sigma \frac{\bar{f}_{ij}}{\bar{r}_{ij}}\right) + \ln w_i^{(1+\alpha)\sigma} - \ln\left(E_j P_j^{\sigma-1}\right)^{1+\alpha}. \tag{A.82}$$

Intensive margin of firm-level exports. The average revenue of firms from country i in country j is $\bar{x}_{ij} \equiv E\left[R_{ij}(\eta)\,|\eta \in \Omega_{ij}\right]$ where $R_{ij}(\eta)$ is given by (A.71). Define the average revenue potential of exporters when n_{ij}% of $i's$ firms in sector s export to j as $\rho_{ij}(n_{ij}) \equiv \frac{1}{n_{ij}}\int_0^{n_{ij}} \rho^m_{ij}(n)\,dn$ where $\rho^m_{ij}(n) \equiv E[r|e = \epsilon_{ij}(n)]$ is the average revenue potential in quantile n of the entry potential distribution. Using the transformation $n = 1 - H^e_{ij}(e)$ such that $e = \epsilon_{ij}(n)$ and $dH^e_{ij}(e) = -dn$, we can follow the same steps as in the baseline model to show that

$$\ln \bar{x}_{ij} - \ln \rho_{ij}(n_{ij}) = \ln \bar{r}_{ij} + \ln w_i^{1-(1+\alpha)\sigma} + \ln\left(E_j P_j^{\sigma-1}\right)^{1+\alpha}. \tag{A.83}$$

Extensive margin of products per firm. The average number of products per firm of i operating in market j is $N^v_{ij} = E\left[N_{ij}(\eta)\,|\eta \in \Omega_{ij}\right]$. The expression for $N_{ij}(\eta)$ in (A.70) implies that

$$N^v_{ij} = \frac{1}{\sigma^\alpha}\left(\bar{r}^N_{ij}\right)^\alpha w_i^{-\alpha\sigma}\left(E_j P_j^{\sigma-1}\right)^\alpha E\left[\left(r^N_{ij}(\eta)\right)^\alpha |\eta \in \Omega_{ij}\right]$$

and, since $\bar{r}_{ij} \equiv \frac{1}{\sigma^\alpha}\left(\bar{r}^N_{ij}\right)^{1+\alpha}$,

$$N^v_{ij} = \sigma^{-\frac{\alpha}{1+\alpha}}\bar{r}_{ij}^{\frac{\alpha}{1+\alpha}} w_i^{-\alpha\sigma}\left(E_j P_j^{\sigma-1}\right)^\alpha E\left[\left(r_{ij}(\eta)\right)^{\frac{\alpha}{1+\alpha}} |\eta \in \Omega_{ij}\right].$$

To obtain the gravity equation, we consider a similar transformation as the one above. Define $\rho^v_{ij}(n_{ij}) \equiv \frac{1}{n_{ij}}\int_0^{n_{ij}} \tilde{\rho}^v_{ij}(n)\,dn$ where $\tilde{\rho}^v_{ij}(n) \equiv E[r^{\frac{\alpha}{1+\alpha}}|e = \epsilon_{ij}(n)]$. Using the transformation $n = 1 - H^e_{ij}(e)$ such that $e = \epsilon_{ij}(n)$ and $dH^e_{ij}(e) = -dn$, we can follow the same steps as in the baseline model to show that

$$\ln N^v_{ij} - \ln \rho^v_{ij}(n_{ij}) = \ln \sigma^{-\frac{\alpha}{1+\alpha}} + \frac{\alpha}{1+\alpha}\ln \bar{r}_{ij} + \ln w_i^{-\alpha\sigma} + \ln\left(E_j P_j^{\sigma-1}\right)^\alpha. \tag{A.84}$$

The elasticity of the average number of varieties per firm with respect to changes in bilateral revenue shifters is $\alpha/(1+\alpha)$, conditional on the composition control function, $\rho^v_{ij}(n_{ij})$, and the origin and destination fixed-effects. Thus, this semiparametric gravity equation can be used to estimate the parameter α. We show below that this parameter is necessary for counterfactual analysis.

A.4.3 General Equilibrium

Part 1. The extensive and intensive margins of firm-level sales, n_{ij} and $\bar{x}_{ij}$, satisfy (A.82) and (A.83) for all i and j. Together with N_i, they determine bilateral trade flows, $X_{ij} = N_i n_{ij} \bar{x}_{ij}$.

Part 2. For all i, total spending, E_i, satisfies (A.79).

Part 3. To derive the labor market clearing condition notice that there are four sources of demand for labor: production of goods, cost of producing new varieties, fixed-cost of entering a market, and fixed-cost of creating a firm. Thus,

$$\begin{array}{rcc} w_i\bar{L}_i & = & \sum_j N_i Pr[\eta \in \Omega_{ij}] \left(1 - \frac{1}{\sigma}\right) E\left[R_{ij}(\eta) \,|\eta \in \Omega_{ij}\right] \\ & + & \sum_j N_i Pr[\eta \in \Omega_{ij}] E\left[C_{ij}^V(\eta) | \eta \in \Omega_{ij}\right] \\ & + & \sum_j N_i Pr[\eta \in \Omega_{ij}] w_i \bar{f}_{ij} E\left[f_{ij}(\eta) \,|\eta \in \Omega_{ij}\right] \\ & + & N_i w_i \bar{F}_i \end{array}$$

From the free entry condition, we know that $w_i\bar{F}_i = \sum_j \mathbb{E}\left[\max\left\{\pi_{ij}(\omega);\ 0\right\}\right]$. Thus,

$$w_i\bar{F}_i = \sum_j Pr[\eta \in \Omega_{ij}] \left(\frac{1}{\sigma} E\left[R_{ij}(\eta) \,|\eta \in \Omega_{ij}\right] - E\left[C_{ij}^V(\eta) | \eta \in \Omega_{ij}\right] - w_i \bar{f}_{ij} E\left[f_{ij}(\eta) \,|\eta \in \Omega_{ij}\right]\right),$$

which implies that

$$w_i\bar{L}_i = \sum_j N_i Pr[\eta \in \Omega_{ij}] E\left[R_{ij}(\eta) \,|\eta \in \Omega_{ij}\right].$$

Thus, since $\bar{x}_{ij} \equiv E\left[R_{ij}(\eta) \,|\eta \in \Omega_{ij}\right]$ and $n_{ij} \equiv Pr[\eta \in \Omega_{ij}]$, this immediately implies

$$w_i\bar{L}_i = \sum_j N_i n_{ij} \bar{x}_{ij}. \tag{A.85}$$

Part 4. Since $p_{ij}(\eta) = \frac{\sigma}{\sigma-1} \frac{\bar{\tau}_{ij} w_i}{\bar{a}_i} \frac{\tau_{ij}(\eta)}{a_i(\eta)}$ for every variety of firm η, the expression for $P_j^{1-\sigma}$ in (A.67) implies that

$$P_j^{1-\sigma} = \sum_i \bar{b}_{ij} \int_{\Omega_{ij}} N_{ij}(\eta) \left(p_{ij}(\eta)\right)^{1-\sigma} \ d\eta.$$

Using the expression for $N_{ij}(\eta)$ in (A.70) and the definitions in (A.72), this expression can be written as

$$P_j^{1-\sigma} = \sum_i w_i^{1-\sigma} \frac{\bar{r}_{ij}}{w_i^{\alpha}} \left[\left(\frac{w_i}{P_j}\right)^{1-\sigma} \frac{E_j}{w_i}\right]^{\alpha} \int_{\Omega_{ij}} r_{ij}(\eta) \ d\eta.$$

Notice that $\int_{\Omega_{ij}} r_{ij}(\eta) \ d\eta = N_i Pr[\eta \in \Omega_{ij}] E[r | \eta \in \Omega_{ij}] = N_i n_{ij} \rho_{ij}(n_{ij})$. This immediately yields

$$P_j^{1-\sigma} = \sum_i \bar{r}_{ij} w_i^{1-\sigma} \left[\left(\frac{w_i}{P_j}\right)^{1-\sigma} \frac{E_j}{w_i}\right]^{\alpha} \rho_{ij}(n_{ij}) n_{ij} N_i,$$

and, therefore,

$$P_j^{1-\sigma} = \sum_i \bar{r}_{ij} w_i^{1-(1+\alpha)\sigma} \left(E_j P_j^{\sigma-1}\right)^{\alpha} \rho_{ij}(n_{ij}) n_{ij} N_i. \tag{A.86}$$

Part 5. From (A.71) and (A.73),

$$\begin{array}{rcl} \mathbb{E}\left[\max\left\{\pi_{ij}(\omega);\ 0\right\}\right] & = & Pr[\eta \in \Omega_{ij}] E\left[\frac{1}{(1+\alpha)\sigma} R_{ij}(\eta) - w_i \bar{f}_{ij} f_{ij}(\eta) | \eta \in \Omega_{ij}\right] \\ & & n_{ij} \left(\frac{1}{(1+\alpha)\sigma} \bar{x}_{ij} - w_i \bar{f}_{ij} E\left[r_{ij}(\eta)/e_{ij}(\eta) | \eta \in \Omega_{ij}\right]\right). \end{array}$$

By defining $e_{ij}^* \equiv (1+\alpha)\sigma \frac{\bar{f}_{ij}}{\bar{r}_{ij}} \frac{w_i^{(1+\alpha)\sigma}}{\left(E_j P_j^{\sigma-1}\right)^{1+\alpha}}$, we can write

$$E\left[r_{ij}(\eta)/e_{ij}(\eta)|\eta \in \Omega_{ij}\right] = \int_{e_{ij}^*}^{\infty} \frac{1}{e}\left[\int_0^{\infty} r dH_{ij}^r(r|e)\right] \frac{dH^e(e)}{1-H^e(e_{ij}^*)}.$$

Consider the transformation $n = 1 - H_{ij}(e)$ such that $e = \bar{\epsilon}_{ij}(n)$. In this case, $dH_{ij}(e) = -dn$ and $n_{ij} = 1 - H_{ij}(e_{ij}^*)$, which implies that

$$E\left[r_{ij}(\omega)/e_{ij}(\omega)|\omega \in \Omega_{ij}\right] = \frac{1}{n_{ij}} \int_0^{n_{ij}} \frac{\rho_{ij}^m(n)}{\epsilon_{ij}(n)} dn.$$

Thus,

$$\mathbb{E}\left[\max\left\{\pi_{ij}(\omega);\ 0\right\}\right] = \frac{1}{(1+\alpha)\sigma} n_{ij}\bar{x}_{ij} - w_i \bar{f}_{ij} \int_0^{n_{ij}} \frac{\rho_{ij}^m(n)}{\epsilon_{ij}(n)}\ dn.$$

Thus, the free entry condition is

$$(1+\alpha)\sigma w_i \bar{F}_i = \sum_j n_{ij}\bar{x}_{ij} - \sum_j \left((1+\alpha)\sigma w_i \bar{f}_{ij}\right) \int_0^{n_{ij}} \frac{\rho_{ij}^m(n)}{\epsilon_{ij}(n)}\ dn.$$

Notice that the summation of (A.82) and (A.83) implies that

$$\ln\left((1+\alpha)\sigma w_i \bar{f}_{ij}\right) = \ln \bar{x}_{ij} - \ln \rho_{ij}(n_{ij}) + \ln \epsilon_{ij}(n_{ij}),$$

which yields

$$(1+\alpha)\sigma w_i \bar{F}_i = \sum_j n_{ij}\bar{x}_{ij} - \sum_j \bar{x}_{ij} \frac{\epsilon_{ij}(n_{ij})}{\rho_{ij}(n_{ij})} \int_0^{n_{ij}} \frac{\rho_{ij}^m(n)}{\epsilon_{ij}(n)}\ dn.$$

By substituting the definition of $\rho_{ij}(n)$, we can write the free entry condition as

$$(1+\alpha)\sigma w_i \bar{F}_i = \sum_j n_{ij}\bar{x}_{ij} - \sum_j n_{ij}\bar{x}_{ij} \frac{\epsilon_{ij}(n_{ij})}{\int_0^{n_{ij}} \rho_{ij}^m(n)\ dn} \int_0^{n_{ij}} \frac{\rho_{ij}^m(n)}{\epsilon_{ij}(n)}\ dn.$$

Using the market clearing condition in (A.80), we have that

$$\frac{1}{N_i} = (1+\alpha)\sigma \frac{\bar{F}_i}{\bar{L}_i} + \sum_j \frac{n_{ij}\bar{x}_{ij}}{w_i \bar{L}_i} \frac{\epsilon_{ij}(n_{ij})}{\int_0^{n_{ij}} \rho_{ij}^m(n)\ dn} \int_0^{n_{ij}} \frac{\rho_{ij}^m(n)}{\epsilon_{ij}(n)}\ dn,$$

which immediately yields

$$N_i = \left[(1+\alpha)\sigma \frac{\bar{F}_i}{\bar{L}_i} + \sum_j \frac{n_{ij}\bar{x}_{ij}}{w_i \bar{L}_i} \frac{\int_0^{n_{ij}} \frac{\rho_{ij}^m(n)}{\epsilon_{ij}(n)}\ dn}{\int_0^{n_{ij}} \frac{\rho_{ij}^m(n)}{\epsilon_{ij}(n_{ij})}\ dn}\right]^{-1}. \tag{A.87}$$

Part 6. The equilibrium vector $\{n_{ij}, \bar{x}_{ij}, E_i, w_i, P_i, N_i\}_{i,j}$ is determined by equations (A.82), (A.83), (A.79), (A.85), (A.86), and (A.87).

A.4.4 Nonparametric Counterfactual Predictions

We now use the equilibrium characterization above to compute counterfactual changes in aggregate outcomes using the functions $\epsilon_{ij}(n)$ and $\rho_{ij}(n)$, as well as σ and α.

1. The extensive and intensive margins of firm-level sales, n_{ij} and $\bar{x}_{ij}$, in (A.82) and (A.83) imply

$$\frac{\epsilon_{ij}(n_{ij}\hat{n}_{ij})}{\epsilon_{ij}(n_{ij})} = \frac{\hat{\bar{f}}_{ij}}{\hat{\bar{r}}_{ij}} \frac{\hat{w}_i^{(1+\alpha)\sigma}}{\left(\hat{E}_j \hat{P}_j^{\sigma-1}\right)^{1+\alpha}}, \tag{A.88}$$

$$\hat{\bar{x}}_{ij} = \hat{\bar{r}}_{ij} \frac{\rho_{ij}(n_{ij}\hat{n}_{ij})}{\rho_{ij}(n_{ij})} \left[\hat{w}_i^{1-(1+\alpha)\sigma} \left(\hat{E}_j \hat{P}_j^{\sigma-1}\right)^{1+\alpha}\right]. \tag{A.89}$$

2. Let $\iota_i \equiv w_i L_i / E_i = \left(\sum_d X_{id}\right) / \left(\sum_o X_{oi}\right)$ be the output-spending ratio in country i in the initial equilibrium. The spending equation in (A.79) implies

$$\hat{E}_i = \iota_i \left(\hat{w}_i \hat{\bar{L}}_i\right) + (1-\iota_i)\hat{\bar{T}}_i, \tag{A.90}$$

3. Let $y_{ij} \equiv (N_i n_{ij} \bar{x}_{ij}) / (w_i L_i) = X_{ij} / \left(\sum_{j'} X_{ij'}\right)$ be the share of i's revenue from sales to j. The labor market clearing condition in (A.85) implies

$$\hat{w}_i \hat{\bar{L}}_i = \sum_j y_{ij} \left(\hat{N}_i \hat{n}_{ij} \hat{\bar{x}}_{ij}\right). \tag{A.91}$$

4. The price index (A.86) implies

$$\hat{P}_j^{1-\sigma} = \sum_i \frac{\bar{r}_{ij} w_i^{1-(1+\alpha)\sigma} \left(E_j P_j^{\sigma-1}\right)^{\alpha} \rho_{ij}(n_{ij}) n_{ij} N_i}{P_j^{1-\sigma}} \left(\hat{\bar{r}}_{ij} \frac{\rho_{ij}(n_{ij}\hat{n}_{ij})}{\rho_{ij}(n_{ij})} \hat{w}_i^{1-(1+\alpha)\sigma} \left(\hat{E}_j \hat{P}_j^{\sigma-1}\right)^{\alpha} \hat{n}_{ij} \hat{N}_i\right).$$

Since $\bar{x}_{ij} = \rho_{ij}(n_{ij}) \bar{r}_{ij} w_i^{1-(1+\alpha)\sigma} \left(E_j P_j^{\sigma-1}\right)^{1+\alpha}$ and $x_{ij} = X_{ij}/E_j = \bar{x}_{ij} n_{ij} N_i / E_j$,

$$\begin{aligned} \hat{P}_j^{1-\sigma} &= \textstyle\sum_i \frac{\bar{x}_{ij} n_{ij} N_i}{E_j} \left(\hat{\bar{r}}_{ij} \frac{\rho_{ij}(n_{ij}\hat{n}_{ij})}{\rho_{ij}(n_{ij})} \hat{w}_i^{1-(1+\alpha)\sigma} \left(\hat{E}_j \hat{P}_j^{\sigma-1}\right)^{\alpha} \hat{n}_{ij} \hat{N}_i\right) \\ &= \textstyle\sum_i x_{ij} \left(\hat{\bar{r}}_{ij} \frac{\rho_{ij}(n_{ij}\hat{n}_{ij})}{\rho_{ij}(n_{ij})} \hat{w}_i^{1-(1+\alpha)\sigma} \left(\hat{E}_j \hat{P}_j^{\sigma-1}\right)^{\alpha} \hat{n}_{ij} \hat{N}_i\right). \end{aligned}$$

Rearranging this expression, we get that

$$\hat{P}_j^{(1-\sigma)(1+\alpha)} = \sum_i x_{ij} \left(\hat{\bar{r}}_{ij} \frac{\rho_{ij}(n_{ij}\hat{n}_{ij})}{\rho_{ij}(n_{ij})} \hat{w}_i^{1-(1+\alpha)\sigma} \hat{E}_j^{\alpha} \hat{n}_{ij} \hat{N}_i\right). \tag{A.92}$$

5. The free entry condition in (A.87) implies

$$N_i \hat{N}_i = \left[(1+\alpha)\sigma \frac{\bar{F}_i}{\bar{L}_i} \frac{\hat{\bar{F}}_i}{\hat{\bar{L}}_i} + \sum_j \frac{n_{ij}\bar{x}_{ij}}{w_i \bar{L}_i} \frac{\hat{n}_{ij}\hat{\bar{x}}_{ij}}{\hat{w}_i \hat{\bar{L}}_i} \frac{\int_0^{n_{ij}\hat{n}_{ij}} \frac{\rho_{ij}^m(n)}{\epsilon_{ij}(n)}\, dn}{\int_0^{n_{ij}\hat{n}_{ij}} \frac{\rho_{ij}^m(n)}{\epsilon_{ij}(n_{ij}\hat{n}_{ij})}\, dn}\right]^{-1}.$$

Using (A.87) to substitute for $(1+\alpha)\sigma\frac{\bar{F}_i}{\bar{L}_i}$,

$$\hat{N}_i = \left[\left(1 - \sum_j y_{ij} \frac{\int_0^{n_{ij}} \frac{\rho_{ij}^m(n)}{\epsilon_{ij}(n)}\, dn}{\int_0^{n_{ij}} \frac{\rho_{ij}^m(n)}{\epsilon_{ij}(n_{ij})}\, d} \right) \frac{\hat{\bar{F}}_i}{\hat{\bar{L}}_i} + \sum_j y_{ij} \frac{\hat{n}_{ij}\hat{\bar{x}}_{ij}}{\hat{w}_i \hat{\bar{L}}_i} \frac{\int_0^{n_{ij}\hat{n}_{ij}} \frac{\rho_{ij}^m(n)}{\epsilon_{ij}(n)}\, dn}{\int_0^{n_{ij}\hat{n}_{ij}} \frac{\rho_{ij}^m(n)}{\epsilon_{ij}(n_{ij}\hat{n}_{ij})}\, dn} \right]^{-1}. \tag{A.93}$$

Thus, the system (A.88)–(A.93) determines the counterfactual predictions in the model. Notice that the system only requires: (i) the elasticity functions $\{\rho_{ij}(n), \epsilon_{ij}(n)\}_{i,j}$, (ii) the elasticities σ and α, and (iii) the data in the initial equilibrium for trade flows and exporter firm shares, $\{X_{ij}, n_{ij}\}_{i,j}$.

A.5 Non-CES Preferences

A.5.1 Environment

Demand. In country j with income y_j, we assume that the Marshallian demand function for product ω can be written as

$$q_j\left(p\left(\omega\right);\boldsymbol{p}_j, y_j\right) = q\left(p\left(\omega\right); P_j\left(\boldsymbol{p}_j, y_j\right), y_j\right) \tag{A.94}$$

where $P_j\left(\boldsymbol{p}_j, y_j\right)$ is a price aggregator and $\boldsymbol{p}_j$ is the vector of all prices in market j. This class of demand functions includes a number of homothetic and non-homothetic examples, as discussed in Arkolakis et al. (2019a) and Matsuyama and Ushchev (2017).

We make two assumptions following Arkolakis et al. (2019a). First, we assume that the demand function features a choke price or, in other words, for each $P_j\left(\boldsymbol{p}, y\right)$ there exists $a \in \mathbb{R}$ such that $q_j\left(x;\boldsymbol{p}_j, y_j\right) = 0$ for all $x \geq a$. This way we can abstract from the fixed cost of entry – that is, we assume that $\bar{f}_{ij} = 0$ for all i and j. Second, we assume that the demand elasticity $\varepsilon_j\left(p\left(\omega\right); P, y\right) = -\partial \ln q\left(p\left(\omega\right); P, y\right)/\partial \ln p$ is decreasing in $p\left(\omega\right)$. For exposition, we suppress the dependence of the demand function and its elasticity on on P and y.

Production. We assume that the production function is

$$C_{ij}(\omega, q) = w_i \frac{\tau_{ij}(\omega)}{a_i(\omega)} \frac{\bar{\tau}_{ij}}{\bar{a}_i} q.$$

Notice that, relative to the baseline, we abstract from the fixed cost of entry. In this case, the extensive margin of firm exports arises from the chock price in demand. We define the firm-specific cost shifter as

$$c_{ij}\left(\omega\right) \equiv \frac{\tau_{ij}\left(\omega\right)}{a_i\left(\omega\right)}$$

Since the production function is constant returns to scale, the quantity for each firm ω can be defined for each pair of markets separately. Thus, given aggregates P and y, the profit maximization problem of firm ω from i when selling in j is

$$\pi_{ij}\left(\omega\right) = \max_{p(\omega)} \left\{ \left(p\left(\omega\right) - w_i \frac{\bar{\tau}_{ij}}{\bar{a}_i} c_{ij}\left(\omega\right) \right) q_{ij}\left(p\left(\omega\right)\right) \right\}$$

The associated first order condition is given by

$$\left(1 - w_i \frac{\bar{\tau}_{ij}}{\bar{a}_i} \frac{c_{ij}\left(\omega\right)}{p_{ij}\left(\omega\right)} \right) = -1/\left(\partial \ln q_j\left(p_{ij}\left(\omega\right)\right)/\partial \ln p\right) = 1/\varepsilon_j\left(p_{ij}\left(\omega\right)\right).$$

Thus, markups are inversely related to the elasticity of demand:

$$m_{ij}(\omega) \equiv \frac{p_{ij}(\omega)}{w_i \frac{\bar{\tau}_{ij}}{\bar{a}_i} c_{ij}(\omega)} = \frac{\varepsilon_j(p_{ij}(\omega))}{\varepsilon_j(p_{ij}(\omega)) - 1}.$$

Furthermore, our second assumption guarantees that the markup is strictly decreasing on the marginal cost, that the price is strictly increasing on marginal cost, and that quantities and sales are strictly decreasing on the marginal cost (see related arguments in Arkolakis et al. (2019a)). This implies that we can perform a change of variables to express all variables in terms of the marginal cost of production:

$$\pi\left(w_i \frac{\bar{\tau}_{ij}}{\bar{a}_i} c_{ij}(\omega)\right) = \left(\frac{m\left(w_i \frac{\bar{\tau}_{ij}}{\bar{a}_i} c_{ij}(\omega)\right) - 1}{m\left(w_i \frac{\bar{\tau}_{ij}}{\bar{a}_i} c_{ij}(\omega)\right)}\right) R\left(w_i \frac{\bar{\tau}_{ij}}{\bar{a}_i} c_{ij}(\omega)\right). \tag{A.95}$$

Since a higher marginal cost lowers markups and sales, the profit function is strictly decreasing on $w_i \frac{\bar{\tau}_{ij}}{\bar{a}_i} c_{ij}(\omega)$. Therefore, given every P and y, there exists a unique revenue potential threshold that determines entry into a market:

$$\omega \in \Omega_{ij} \Leftrightarrow w_i \frac{\bar{\tau}_{ij}}{\bar{a}_i} c_{ij}(\omega) \leq c_j^*(P_j, y_j) \quad \text{such that} \quad \pi\left(c_j^*(P_j, y_j); P_j, y_j\right) = 0. \tag{A.96}$$

Conditional on entering, the revenues and profits are

$$R_{ij}(\omega) = R_{ij}\left(w_i \frac{\bar{\tau}_{ij}}{\bar{a}_i} c_{ij}(\omega); P_j, y_j\right) \quad \text{and} \quad \pi_{ij}(\omega) = \pi\left(w_i \frac{\bar{\tau}_{ij}}{\bar{a}_i} c_{ij}(\omega); P_j, y_j\right). \tag{A.97}$$

Entry. Let us assume that firms pay a fixed entry cost $\bar{F}_i$ in domestic labor to get a draw of variety characteristics from a distribution:

$$v_i(\omega) = \{a_i(\omega), \tau_{ij}(\omega)\}_j \sim G_i(v) \tag{A.98}$$

In expectation, firms only pay the fixed entry cost if ex-ante profits exceed entry them:

$$\sum_j E\left[\max\{\pi_{ij}(\omega);\ 0\}\right] = w_i \bar{F}_i, \tag{A.99}$$

Market clearing. We follow Dekle et al. (2008) by introducing exogenous international transfers, so that spending is

$$E_i = w_i \bar{L}_i + \bar{T}_i, \quad \sum_i \bar{T}_i = 0. \tag{A.100}$$

Since labor is the only factor of production, labor income in i equals the total revenue of firms from i:

$$y_i = w_i \bar{L}_i = \int_{\omega \in \Omega_{ij}} R_{ij}\left(w_i \frac{\bar{\tau}_{ij}}{\bar{a}_i} c_{ij}(\omega); P, y\right). \tag{A.101}$$

Equilibrium. Given the distribution in (A.98) , the equilibrium is the vector $\{P_i, y_i, \{\Omega_{ij}\}_j, N_i, E_i, w_i\}_i$ satisfying (A.94), (A.96), (A.99), (A.100), (A.101) for all i.

A.5.2 Extensive and Intensive margin of Firm-level Export

We now turn to the characterization of the semiparametric gravity equations. We consider the distribution of firm-specific shifts of marginal costs implies by G_i:

$$c_{ij}(\omega) \sim H_{ij}(c), \tag{A.102}$$

where H_{ij} has full support in $\mathbb{R}_+$.

Extensive margin of firm-level exports. The share of firms of country i serving market j is $n_{ij} = Pr\left[\omega \in \Omega_{ij}\right] = Pr\left[c_{ij}(\omega) \leq \frac{\bar{a}_i}{\bar{\tau}_{ij} w_i} c_j^*\left(P_j, y_j\right)\right]$.

$$n_{ij} = H_{ij}\left(\frac{\bar{a}_i}{\bar{\tau}_{ij} w_i} c_j^*\left(P_j, y_j\right)\right)$$

We define $\epsilon_{ij}(n) \equiv \left(H_{ij}\right)^{-1}(n)$. Notice that it is now strictly increasing in n. Thus,

$$\ln \epsilon_{ij}(n_{ij}) = \ln\left(\bar{a}_i / \bar{\tau}_{ij}\right) - \ln w_i + \ln c_j^*\left(P_j, y_j\right). \tag{A.103}$$

This is the semiparametric gravity equation for the extensive margin of firm exports.

Intensive margin of firm-level exports. As before, the average revenue of firms from country i in country j is $\bar{x}_{ij} \equiv E\left[R_{ij}(\omega) | \omega \in \Omega_{ij}\right]$. Thus,

$$\bar{x}_{ij} = \int_0^{\frac{\bar{a}_i}{\bar{\tau}_{ij} w_i} c_j^*(P_j, y_j)} R_{ij}\left(w_i \frac{\bar{\tau}_{ij}}{\bar{a}_i} c; P_j, y_j\right) \frac{dH_{ij}(c)}{H_{ij}\left(\frac{\bar{a}_i}{\bar{\tau}_{ij} w_i} c_j^*\left(P_j, y_j\right)\right)}$$

We can then use the transformation, $n = H_{ij}(c)$ such that $dn = dH_{ij}(c)$, $c = \epsilon_{ij}(n)$, and $n_{ij} = H_{ij}\left(\frac{\bar{a}_i}{\bar{\tau}_{ij} w_i} c_j^*\left(P_j, y_j\right)\right)$. Thus,

$$\bar{x}_{ij} = \frac{1}{n_{ij}} \int_0^{n_{ij}} R_{ij}\left(w_i \frac{\bar{\tau}_{ij}}{\bar{a}_i} \epsilon_{ij}(n); P_j, y_j\right) dn.$$

Using (A.103),

$$\bar{x}_{ij} = \frac{1}{n_{ij}} \int_0^{n_{ij}} R_{ij}\left(c_j^*\left(P_j, y_j\right) \frac{\epsilon_{ij}(n)}{\epsilon_{ij}(n_{ij})}; P_j, y_j\right) dn. \tag{A.104}$$

In this case, we can derive an expression for average sales as function of $\epsilon_{ij}(n)$. So, although we do not have a gravity equation for average firm exports, this is entirely determined by the function governing the semiparametric gravity equation for the extensive margin of firm exports.

For this extension, we do not derive the model's general equilibrium predictions because it requires specifying the price index aggregator of demand, $P_j\left(\boldsymbol{p}_j, y_j\right)$. Conditional on knowing the price index function, we can follow the same steps as in our baseline model to characterize the price index in terms of the distribution of firm firm-specific marginal cost shifters, $H_{ij}(c)$, and, therefore, in terms of the extensive margin elasticity function, $\epsilon_{ij}(n) \equiv \left(H_{ij}\right)^{-1}(n)$.

B Additional Results: Multiple Country Groups

Our baseline estimates impose that the elasticity functions are identical for all exporter-destination pairs ($G = 1$). In this section, we estimate alternative specifications where we allow the elasticity function to vary across groups of countries.

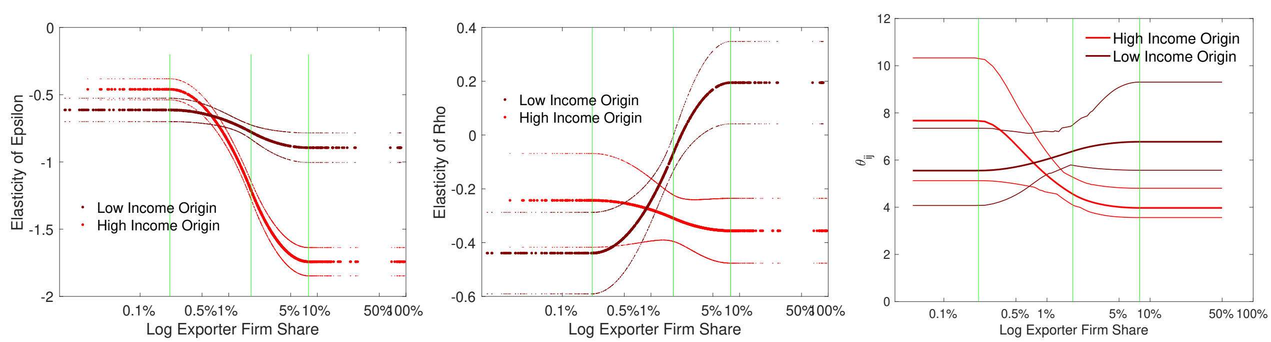
B.1 Heterogeneity with respect to per capita income

We first implement our estimation procedure with country groups defined defined in terms of per capita GDP. This type of heterogeneity in trade elasticity has been explored by Adao et al. (2017). We use a cutoff of \$9,000 of per capita GDP in 2002 to divide our sample into developed and developing nations. Column (6) of Table OA.1 in Appendix B.1 shows the list of developed and developing countries in our sample.

Figure SM.1 reports estimates for two groups defined in terms of development of the origin country. Panel (a) indicates that the extensive margin elasticity varies less with the exporter firm share in developing origin countries. In addition, Panel (b) indicates that, in developing countries, selection forces are weaker for high levels of n_{ij}. This translates into a roughly constant trade elasticity of six for developing countries. For developed countries, our estimates are similar to the baseline results in Figure 3. This reflects the fact that developed origin countries constitute the majority of our sample.

We also implement our estimation procedure for four groups defined in terms of per capita income of both the origin and destination countries. Figure SM.2 shows that the per capita income of the destination country does not have a large impact on the estimates reported in Figure SM.1.

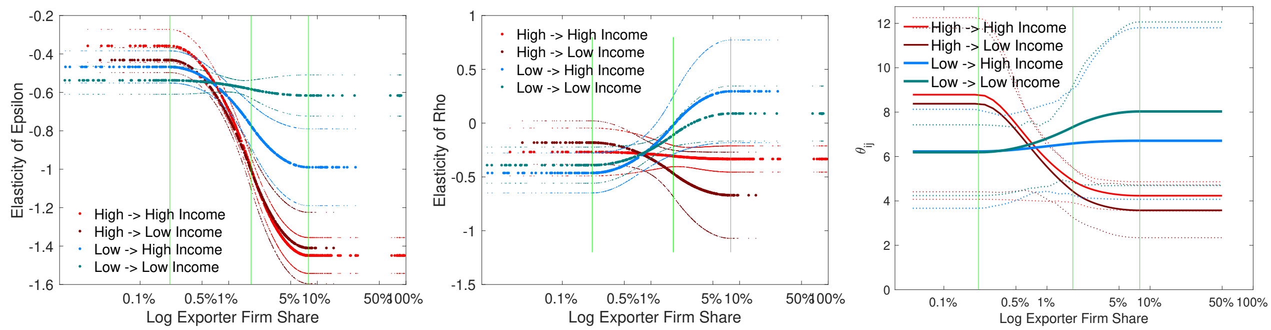
Figure SM.1: Semiparametric gravity estimation – Country groups defined in terms of per capita income of origin country



(a) Elasticity of $\epsilon(n)$ (b) Elasticity of $\rho(n)$ (c) Implied $\theta(n)$

Note. Estimates obtained with GMM estimator in (33). Estimates obtained with a cubic spline over three intervals ($K = 3$) for a two groups ($G = 2$). Groups defined in terms of origin country per capita income – see column (6) of Table OA.1 of Appendix B.1. Calibration of $\tilde{\sigma} = 2.9$. Thick lines are the point estimates and thin lines are the 95% confidence intervals computed with robust standard errors for $\epsilon(n)$ and $\rho(n)$ and 1,000 bootstrap draws for $\theta(n)$.

Figure SM.2: Semiparametric gravity estimation – Country groups defined in terms of per capita income of origin and destination countries



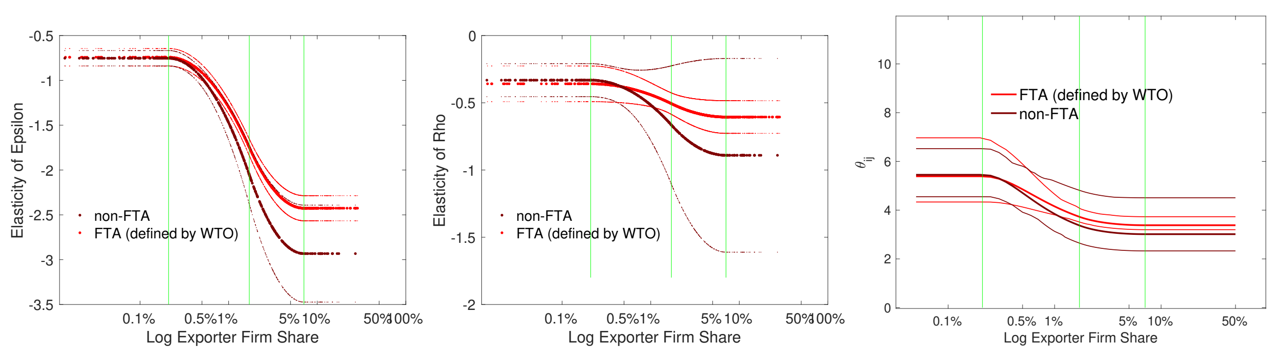
(a) Elasticity of $\epsilon(n)$ (b) Elasticity of $\rho(n)$ (c) Implied $\theta(n)$

Note. Estimates obtained with GMM estimator in (33). Estimates obtained with a cubic spline over three intervals ($K = 3$) for a four groups ($G = 4$). Groups defined in terms of per capita income of origin and destination countries – see column (6) of Table OA.1 of Appendix B.1. Calibration of $\tilde{\sigma} = 2.9$. Thick lines are the point estimates and thin lines are the 95% confidence intervals computed with robust standard errors for $\epsilon(n)$ and $\rho(n)$ and 100 bootstrap draws for $\theta(n)$.

B.2 Heterogeneity with respect to bilateral attributes - Free Trade Areas

We implement our estimation procedure for two groups of exporter-importer pairs defined as country pairs inside and outside a common Free Trade Areas (FTA) (using the CEPII bilateral gravity dataset). A large body of literature has documented that membership in free trade areas reduces trade costs – see Head and Mayer (2014). We investigate here if it also affects the different elasticity margins of trade flows. Figure SM.3 indicates that there are only small differences in the estimates for countries inside and outside common free trade areas.

Figure SM.3: Semiparametric gravity estimation – Country groups defined in terms of membership in free trade areas



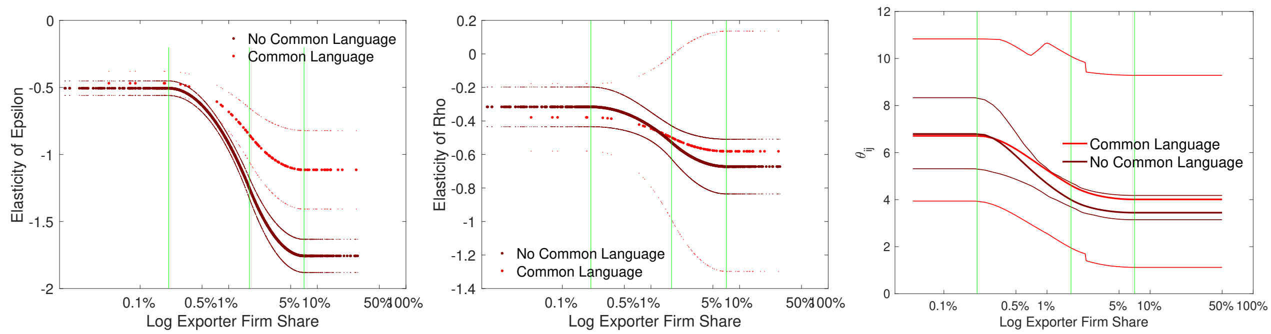
(a) Elasticity of $\epsilon(n)$ (b) Elasticity of $\rho(n)$ (c) Implied $\theta(n)$

Note. Estimates obtained with GMM estimator in (33) in the 2012 sample of 1,443 origin-destination pairs described in Table OA.1 of Appendix B.1. Estimates obtained with a cubic spline over three intervals ($K = 3$) for a two groups ($G = 2$). Groups defined as country pairs inside and outside a common Free Trade Areas (FTA). Calibration of $\tilde{\sigma} = 2.9$. Thick lines are the point estimates and thin lines are the 95% confidence intervals computed with robust standard errors for $\epsilon(n)$ and $\rho(n)$ and 1,000 bootstrap draws for $\theta(n)$.

B.3 Heterogeneity with respect to bilateral attributes - Common Languages and Currency

Figures SM.4 and SM.5 investigate whether the elasticity functions differ across country pairs that share a common language or currency. The literature has documented that these two characteristics are associated with higher bilateral trade flows – see Head and Mayer (2014). We again find no evidence that such characteristics affect the elasticity functions.

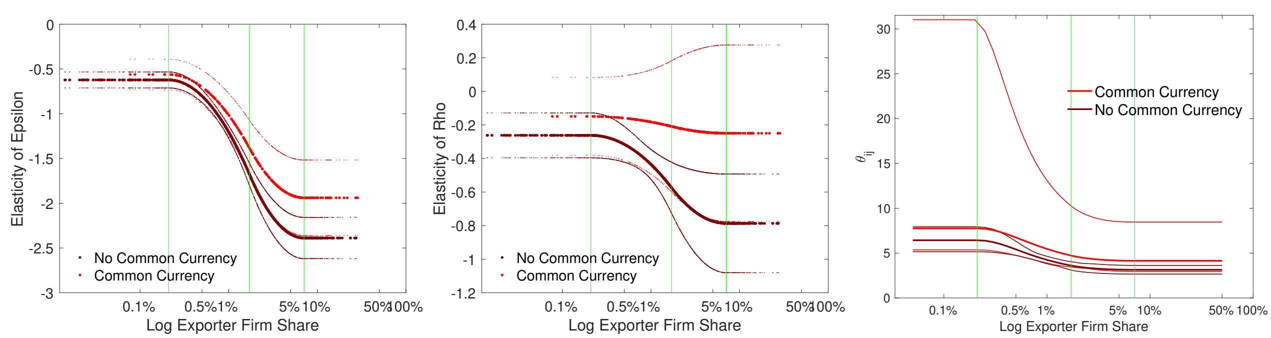
Figure SM.4: Semiparametric gravity estimation – Country groups based on having a common language



(a) Elasticity of $\epsilon(n)$ (b) Elasticity of $\rho(n)$ (c) Implied $\theta(n)$

Note. Estimates obtained with GMM estimator in (33) in the 2012 sample of 1,443 origin-destination pairs described in Table OA.1 of Appendix B.1. Estimates obtained with a cubic spline over three intervals ($K = 3$) for a two groups ($G = 2$). Country groups defined in terms of having at least 9% of a country speak the same language. Calibration of $\tilde{\sigma} = 2.9$. Thick lines are the point estimates and thin lines are the 95% confidence intervals computed with robust standard errors for $\epsilon(n)$ and $\rho(n)$ and 1,000 bootstrap draws for $\theta(n)$.

Figure SM.5: Semiparametric gravity estimation – Country groups based on having a common currency



(a) Elasticity of $\epsilon(n)$ (b) Elasticity of $\rho(n)$ (c) Implied $\theta(n)$

Note. Estimates obtained with GMM estimator in (33) in the 2012 sample of 1,443 origin-destination pairs described in Table OA.1 of Appendix B.1. Estimates obtained with a cubic spline over three intervals ($K = 3$) for a two groups ($G = 2$). Country groups defined in terms of sharing an official currency. Calibration of $\tilde{\sigma} = 2.9$. Thick lines are the point estimates and thin lines are the 95% confidence intervals computed with robust standard errors for $\epsilon(n)$ and $\rho(n)$ and 100 bootstrap draws for $\theta(n)$.

C Computation Algorithms

C.1 Hat Algebra

We now describe an algorithm to compute the changes in aggregate outcomes that solve the system in Appendix A.2 for an arbitrary trade cost change from $\{\bar{\tau}_{ij}\}_{ij}$ to $\{\bar{\tau}'_{ij}\}_{ij}$.

1. Compute the partition of the shock with length R: $d\ln\bar{\tau}_{ij} = \frac{1}{R}\left(\ln\bar{\tau}'_{ij} - \ln\bar{\tau}_{ij}\right)$. Consider the initial equilibrium with $\theta_{ij}(n^0_{ij})$ and $\{x^0_{ij}, y^0_{ij}, \iota^0_j\}$.
2. For each step r, we consider the initial conditions $\left(\varepsilon_{ij}(n^{r-1}_{ij}), \varrho_{ij}(n^{r-1}_{ij}), \theta_{ij}(n^{r-1}_{ij})\right)$ and $\{x^{r-1}_{ij}, y^{r-1}_{ij}, \iota^{r-1}_j\}$.

   (a) Compute $(d\ln\boldsymbol{\tau}^{w,r}, d\ln\boldsymbol{\tau}^{p,r})$ and $d\ln\boldsymbol{w}^r$ using (OA.38) (for a given numerarie with $d\ln w^r_m = 0$).

   (b) Solve $d\ln\boldsymbol{P}^r$ using (OA.37).

   (c) Solve $d\ln n^r_{ij}$ and $d\ln\bar{x}^r_{ij}$ using (OA.14) and (OA.15).

   (d) Solve for $d\ln N^r_i$ using (OA.24).

   (e) Compute the change in bilateral trade flows: $d\ln X^r_{ij} = d\ln\bar{x}^r_{ij} + d\ln n^r_{ij} + d\ln N^r_i$.

   (f) Compute the initial conditions for the next step: $X^r_{ij} = X^{r-1}_{ij}e^{d\ln X^r_{ij}}$ and $n^r_{ij} = n^{r-1}_{ij}e^{d\ln n^r_{ij}}$.

   (g) Compute $x^r_{ij} = X^r_{ij}/\sum_o X^r_{oj}$, $y^r_{ij} = X^r_{ij}/\sum_d X^r_{id}$, $\iota^r_i = \left(\sum_d X^r_{id}\right)/\left(\sum_o X^r_{oi}\right)$, and $\left(\varepsilon_{ij}(n^r_{ij}), \varrho_{ij}(n^r_{ij}), \theta_{ij}(n^r_{ij})\right)$.

3. Compute changes in aggregate variables as

$$\hat{\boldsymbol{w}}^{linear} = \exp\left(\sum_{r=1}^{R} d\ln\boldsymbol{w}^r\right), \quad \hat{\boldsymbol{P}}^{linear} = \exp\left(\sum_{r=1}^{R} d\ln\boldsymbol{P}^r\right), \quad \hat{\boldsymbol{N}}^{linear} = \exp\left(\sum_{r=1}^{R} d\ln\boldsymbol{N}^r\right).$$

4. Use $\{\hat{w}^{linear}_i, \hat{P}^{linear}_i\}$ as an initial guess is the solution of the hat algebra system. Set the same numerarie as above, $\hat{w}_m \equiv 1$.

   (a) Given guess of $(\hat{\boldsymbol{w}}, \hat{\boldsymbol{P}})$, compute

$$\frac{\epsilon_{ij}(n_{ij}\hat{n}_{ij})}{\epsilon_{ij}(n_{ij})} = \left(\hat{\bar{\tau}}_{ij}\right)^{\sigma-1}\left[\left(\frac{\hat{w}_i}{\hat{P}_j}\right)^{\sigma}\frac{\hat{P}_j}{\iota_j\hat{w}_j}\right],$$

$$\hat{\bar{x}}_{ij} = \left(\hat{\bar{\tau}}_{ij}\right)^{1-\sigma}\frac{\rho_{ij}(n_{ij}\hat{n}_{ij})}{\rho_{ij}(n_{ij})}\left[\left(\frac{\hat{w}_i}{\hat{P}_j}\right)^{1-\sigma}\iota_j\hat{w}_j\right].$$

   From (OA.13),

$$\hat{N}_i = \left[1 - \sum_j y_{ij}\left(\frac{\int_0^{n_{ij}}\frac{\rho^m_{ij}(n)}{\epsilon_{ij}(n)}\,dn}{\int_0^{n_{ij}}\frac{\rho^m_{ij}(n)}{\epsilon_{ij}(n_{ij})}\,dn}\right) + \sum_j y_{ij}\frac{\hat{n}_{ij}\hat{\bar{x}}_{ij}}{\hat{w}_i}\left(\frac{\int_0^{n_{ij}\hat{n}_{ij}}\frac{\rho^m_{ij}(n)}{\epsilon_{ij}(n)}\,dn}{\int_0^{n_{ij}\hat{n}_{ij}}\frac{\rho^m_{ij}(n)}{\epsilon_{ij}(n_{ij}\hat{n}_{ij})}\,dn}\right)\right]^{-1}.$$

   (b) Compute the functions:

$$F^P_j\left(\hat{\boldsymbol{w}}, \hat{\boldsymbol{P}}\right) \equiv \hat{P}_j^{1-\sigma} - \sum_i x_{ij}\left(\left(\hat{\bar{\tau}}_{ij}\right)^{1-\sigma}\frac{\rho_{ij}(n_{ij}\hat{n}_{ij})}{\rho_{ij}(n_{ij})}\hat{w}_i^{1-\sigma}\hat{n}_{ij}\hat{N}_i\right)$$

$$F_i^w\left(\hat{\boldsymbol{w}}, \hat{\boldsymbol{P}}\right) \equiv \hat{w}_i - \sum_j y_{ij}\left(\hat{N}_i \hat{n}_{ij} \hat{\hat{x}}_{ij}\right).$$

(c) Find $\left(\hat{\boldsymbol{w}}, \hat{\boldsymbol{P}}\right)$ that minimizes $\left\{|F_j^P\left(\hat{\boldsymbol{w}}, \hat{\boldsymbol{P}}\right)|, |F_i^w\left(\hat{\boldsymbol{w}}, \hat{\boldsymbol{P}}\right)|\right\}$.

C.2 Gains from trade

We now describe an algorithm to compute the gains from trade described in Appendix A.6.

1. Define the uni-dimensional function

$$F_i(\hat{n}_{ii}^A) = \sum_j y_{ij}\left(1 - \frac{\int_0^{n_{ij}} \frac{\rho_{ij}^m(n)}{\epsilon_{ij}(n)}\, dn}{\int_0^{n_{ij}} \frac{\rho_{ij}^m(n)}{\epsilon_{ij}(n_{ij})}\, dn}\right) - \frac{1}{\hat{N}_i^A(\hat{n}_{ii}^A)} \frac{y_{ii}}{x_{ii}}\left(1 - \frac{\int_0^{n_{ii}\hat{n}_{ii}^A} \frac{\rho_{ii}^m(n)}{\epsilon_{ii}(n)}\, dn}{\int_0^{n_{ii}\hat{n}_{ii}^A} \frac{\rho_{ii}^m(n)}{\epsilon_{ii}(n_{ii}\hat{n}_{ii}^A)}\, dn}\right)$$

where

$$\hat{N}_i^A(\hat{n}_{ii}^A) = \frac{\rho_{ii}(n_{ii})}{\epsilon_{ii}(n_{ii})} \frac{\epsilon_{ii}\left(n_{ii}\hat{n}_{ii}^A\right)}{\rho_{ii}\left(n_{ii}\hat{n}_{ii}^A\right)} \frac{1}{\hat{n}_{ii}^A} \frac{1}{x_{ii}}.$$

2. For each i, we find $\hat{n}_{ii}^A$ such that $F_i(\hat{n}_{ii}^A) = 0$. We then compute the gains from trade using equation (23).